MANONMANIAM SUNDARANAR UNIVERSITY

TIRUNELVELI

DIRECTORATE OF DISTANCE AND CONTINUINING EDUCATION

B.Sc Mathematics

Mathematics For Competitive Examinations II

(JSMA21)

MATHEMATICS FOR COMPETITIVE EXAMINATIONS II

(JSMA21)

UNIT I

Simple Interest and Compound Interest

UNIT II

Time and Work

UNIT III

Time and Distance

UNIT IV

Chain Rule

UNIT V

Pipes and Cistern

Recommended Text

R. S. Agarwal – Objective Arithmetic, Published by S Chand and Co. Ltd, Edition(2018).

UNIT I

SIMPLE INTEREST AND COMPOUND INTEREST

SIMPLE INTEREST

1. Principal : The money borrowed or lent out for a certain period is called the principal or the sum.
2. Interest : Extra money paid for using other's money is called interest.
3. Simple Interest (S.I) : If the interest on a sum borrowed for a certain period is reckoned uniformly, then is called simple interest.

   Let Principal = P, Rate =R% per annum (p.a) and Time = T years. Then,

   (i) $S.I = \left(\frac{P \times R \times T}{100}\right)$

   (ii) $P = \left(\frac{100 \times S.I}{R \times T}\right); R = \left(\frac{100 \times S.I}{P \times T}\right) \; and \; T = \left(\frac{100 \times S.I}{P \times R}\right)$

SOLVED PROBLEMS

Example 1. Find the simple interest on Rs. 68,000 at $16\frac{2}{3}\%$ per annum for 9 months.

Solution:

$$P = Rs.68000, R = \frac{50}{3}\% \; p.a \; and \; T = \frac{9}{12} \; years = \frac{3}{4} \; years.$$

$$\therefore S.I. = \left(\frac{P \times R \times T}{100}\right) = Rs.\left(68000 \times \frac{50}{3} \times \frac{3}{4} \times \frac{1}{100}\right) = Rs.8500.$$

Example 2. Find the simple interest on $Rs.3000 \; at \; 6\frac{1}{4}\%$ per annum for the period from 4th Feb., 2005 to 18th April, 2005.

Solution:

$$Time = (24 + 31 - 18) days = 73 \; days = \frac{73}{365} year = \frac{1}{5} year.$$

$$P = Rs.3000 \; and \; R = 6\frac{1}{4}\% \; p.a = \frac{25}{4}\% \; p.a.$$

$$\therefore S.I. = Rs.\left(3000 \times \frac{25}{4} \times \frac{1}{5} \times \frac{1}{100}\right) = Rs.37.50.$$

Remark: The day on which money is deposited is not counted while the day on which money is withdrawn in counted.

Example 3. A sum at simple interest at $13\frac{1}{2}\%$ per annum amounts to $Rs.2502.50$ after 4 years. Find the sum.

Solution:

$$Let\ sum\ be\ Rs.x.Then\ sum = Rs.\left(x \times \frac{27}{2} \times 4 \times \frac{1}{100}\right) = Rs.\frac{27x}{50}.$$

$$\therefore Amount = Rs.\left(x + \frac{27x}{50}\right) = Rs.\frac{77x}{50}.$$

$$\therefore \frac{77x}{50} = 2502.50 \Leftrightarrow x = \frac{2502.50 \times 50}{77} = 1625.$$

$$Hence, sum = Rs.1625.$$

Example 4. A sum of Rs. 800 amounts to Rs. 920 in 3 years at simple interest. If the interest rate is increased by 3%, it would amount to how much?

Solution:

$$S.I = rs.(920 - 800) = Rs.120; P = Rs.800, T = 3\ yrs.$$

$$\therefore R = \left(\frac{100 \times 120}{800 \times 3}\right)\% = 5\%.$$

$$New\ rate = (5 + 3)\% = 8\%.$$

$$New\ S.I. = Rs.\left(\frac{800 \times 8 \times 3}{100}\right) = Rs.192.$$

$$\therefore New\ amount = Rs.(800 + 192) = Rs.992.$$

Example 5. Adam borrowed some money at the rate of 6% p.a. for the first two years, at the rate of 9% p.a. for the next three years, and at the rate of 14% p.a. for the period beyond five years. If he pays a total interest of Rs. 11,400 ate the end of nine years, how much money did he borrow?

Solution:

Let the sum borrowed be x. Then,

$$\left(\frac{x \times 6 \times 2}{100}\right) + \left(\frac{x \times 9 \times 3}{100}\right) + \left(\frac{x \times 14 \times 4}{100}\right) = 11400$$

$$\Leftrightarrow \left(\frac{3x}{25} + \frac{27x}{100} + \frac{14x}{25}\right) = 11400 \Leftrightarrow \frac{95x}{100} = 11400$$

$$\Leftrightarrow x = \left(\frac{11400 \times 100}{95}\right) = 12000.$$

Hence, sum borrowed = Rs. 12,000.

Example 6. A certain sum of money amounts to Rs. 1008 in 2 years and to Rs. 1164 in $3\frac{1}{2}$ years. Find the sum and the rate of interest.

Solution:

S.I. for $1\frac{1}{2}$ years $= Rs.(1164 - 1008) = Rs.156.$

$S.I.\ for\ 2\ years = Rs.\left(156\ \times\frac{2}{3}\times 2\right) = Rs.208.$

$\therefore Principal = Rs.(1008 - 208) = Rs.800.$

Now, $P = 800, T = 2\ and\ S.I. = 208.$

$\therefore Rate = \left(\frac{100\times208}{800\times2}\right)\% = 13\%.$

Example 7. At what rate percent per annum will a sum of money double in 16 years?

Solution:

Let principal = P. Then S.I. = P and T=16 yrs.

$\therefore Rate = \left(\frac{100\times P}{P\times16}\right)\% = 6\frac{1}{4}\%\ p.a.$

Example 8. The simple interest on a sum of money is $\frac{4}{9}$ of the principle. Find the rate percent and time, if both are numerically equal.

Solution:

Let sum = Rs. X. Then, S.I. = Rs. $\frac{4x}{9}$.

Let rate = R% and time = R years.

Then, $\left(\frac{x\times R\times R}{100}\right) = \frac{4x}{9}\ or\ R^2 = \frac{400}{9}\ or\ R = \frac{20}{3} = 6\frac{2}{3}.$

$\therefore Rate = 6\frac{2}{3}\%$ and Time $= 6\frac{2}{3}\%\ yrs = 6\ yrs\ 8\ months.$

Example 9. The simple interest on a certain sum of money for $2\frac{1}{2}$ years at 12% per annum is Rs. 40 less than the simple interest on the same sum for $3\frac{1}{2}$ years at 10% per annum. Find the sum.

Solution:

Let the sum be Rs. x. Then, $\left(\frac{x\times10\times7}{100\times2}\right) - \left(\frac{x\times12\times5}{100\times2}\right) = 40$

$\Leftrightarrow \frac{7x}{20} - \frac{3x}{10} = 40 \Leftrightarrow x = (40\times20) = 800.$

Hence, the sum is Rs. 800.

Example 10. A sum was put at simple interest at a certain rate for 3 years. Had it been put at 2% higher rate, it would have fetched Rs. 360 more. Find the sum.

Solution:

Let sum = P and original rate = R. Then, $\left[\frac{P\times(R+2)\times3}{100}\right]-\left[\frac{P\times R\times3}{100}\right]=360.$

$\Leftrightarrow 3PR+6P-3PR=36000 \Leftrightarrow 6P=36000 \Leftrightarrow P=6000.$

Hence, sum = Rs. 6000.

Example 11. What annual instalment will discharge a debt of Rs. 1092 due in 3 years at 12% simple interest?

Solution:

Let each instalment be Rs. x. Then, $\left(x+\frac{x\times12\times1}{100}\right)+\left(x+\frac{x\times12\times2}{100}\right)+x=1092$

$\Leftrightarrow \frac{28x}{25}+\frac{31x}{25}+x=1092 \Leftrightarrow (28x+31x+25x)=(1092\times25)$

$\Leftrightarrow x=\left(\frac{1092\times25}{84}\right)=325.$

$\therefore$ Each instalment = Rs. 325.

Example 12. A sum of Rs. 1550 is lent out into two parts, one at 8% and another one at 6%. If the total annual income is Rs. 106, find the money lent at each rate.

Solution:

Let the sum lent at 8% be Rs. x and that at 6% be Rs. (1550 – x).

$\therefore \left[\frac{x\times8\times1}{100}\right]+\left[\frac{(1550-x)\times6\times1}{100}\right]=106$

$\Leftrightarrow 8x+9300-6x=10600 \Leftrightarrow 2x=1300 \Leftrightarrow x=650.$

$\therefore$ Money lent at 8% = Rs. 650. Money lent at 6% = Rs. (1550 – 650) = Rs. 900.

EXERCISE

1. At the rate of $8\frac{1}{2}\%p.a.$ simple interest, a sum of Rs. 4800 will earn how much interest in 2 years 3 months?
2. What will be the simple interest earned on an amount of Rs. 16,800 in 9 months at the rate of $6\frac{1}{4}\%\ p.a.$?

3. The simple interest on Rs. 1820 from March 9, 2003 to May 21, 2003 at $7\frac{1}{2}\%$ rate will be:
4. A person borrows Rs. 5000 for 2 years at 4% p.a. simple interest. He immediately lends it to another person at $6\frac{1}{4}\%$ p.a. for 2 years. Find the gain in the transaction per year.
5. How much time will it take for an amount of Rs. 450 to yield Rs. 81 as interest at 4.5% per annum of simple interest?
6. A sum of Rs. 12,500 amounts to Rs. 15,500 in 4 years at the rate of simple interest. What is the rate of interest?
7. A sum of Rs. 1600 gives a simple interest of Rs. 252 in 2 years and 4 months. The rate of interest per annum is:
8. Reena took a loan of Rs. 1200 with simple interest for as many years as the rate of interest. If she paid Rs. 432 as interest at the end of the loan period, what was the rate of interest?
9. A man took a loan from a bank at the rate of 12% p.a. simple interest. After 3 years he had to pay Rs. 5400 interest only for the period. What is the principal amount borrowed by him?
10. What is the present worth of Rs. 132 due in 2 years at 5% simple interest per annum?
11. A sum fetched a total simple interest of Rs. 4016.25 at the rate of 9 p.c.p.a in 5 years. What is the sum?
12. The simple interest at x% for x years will be Rs. x on a sum of:
13. Rs. 800 becomes Rs. 956 in 3 years at a certain rate of simple interest. If the rate of interest is increased by 4%, what amount will Rs. 800 become in 3 years?
14. A certain amount earns simple interest of Rs. 1750 after 7 years. Had the interest been 2% more, how much more interest would it have earned?
15. In how many years, Rs. 150 will produce the same interest @ 8% as Rs. 800 produce in 3 years @ $4\frac{1}{2}\%$?
16. If Rs. 64 amounts to Rs. 83.20 in 2 years, what will Rs. 86 amount to in 4 years at the same rate percent per annum?
17. The simple interest on a certain sum of money at the rate of 5% p.a. for 8 years is Rs. 840. At what rate of interest the same amount of interest can be received on the same sum after 5 years?

18. The interest on a certain deposit at 4.5% p.a. is Rs. 202.50 in one year. How much will the additional interest in one year be on the same deposit at 5% p.a.?
19. A sum invested at 5% simple interest per annum grows to Rs, 504 in 4 years. The same amount at 10% simple interest per annum in $2\frac{1}{2}$ years will grow to;
20. What will be the ratio of simple interest earned by certain amount at the same rate of interest for 6 years and that for 9 years?
21. Nitin borrowed some money at the rate of 6% p.a. for the first three years, 9% p.a. for the next five years and 13% p.a. for the period beyond eight years. If the total interest paid by him at the end of eleven years is Rs. 8160, how much money did he borrow?
22. The simple interest on a sum of money will be Rs. 600 after 10 years. If the principal is trebled after 5 years, what will be the total interest at the end of the tenth year?
23. The simple interest on Rs. 10 for 4 months at the rate of 3 paise per rupee per month is?
24. An automobile financier claims to be lending money at simple interest, but he includes the interest every six months for calculating the principal. If he is charging an interest of 10%, the effective rate of interest becomes?
25. A sum of money at simple interest amounts to Rs. 815 in 3 years and to Rs. 854 in 4 years. What is the sum?
26. A sum of money lent out at simple interest amounts to Rs. 720 after 2 years and to Rs. 1020 after a further period of 5 years. What is the sum?
27. A sum of money amounts to Rs. 9800 after 5 years and Rs. 12005 after 8 years at the same rate of simple interest. What is the rate of interest per annum?
28. A certain sum of money at simple interest amounts to Rs. 1012 in $2\frac{1}{2}$ years and to Rs. 1067.20 in 4 years. What is the rate of interest per annum?
29. In how many years will a sum of money double itself at 12% per annum?
30. At what rate percent of simple interest will a sum of money double itself in 12 years?
31. The rate at which a sum becomes four times of itself in 15 years at S.I., will be?
32. If a sum of money at simple interest doubles in 6 years, it will become 4 times in:
33. A sum of money trebles itself in 15 years 6 months. In how many years would it double itself?
34. Consider the following statements:

    If a sum of money is lent at simple interest, then the

1. Money gets doubled in 5 years if the rate of interest is $16\frac{2}{3}$ %
2. Money gets doubled in 5 years if the rate of interest is 20%
3. Money becomes four times in 10 years if it gets doubled in 5 years.

Of these satements,

35. The simple interest on a sum of money at 8% per annum for 6 years is half the sum. What is the sum?
36. At what rate percent per annum will the simple interest on a sum of money be $\frac{2}{5}$ of the amount in 10 years?
37. In how much time would the simple interest on a certain sum be 0.125 times the principal at 10% per annum?
38. How long will it take a sum of money invested at 5% p.a. S.I. to increase its value by 40%?
39. A sum of money becomes $\frac{7}{6}$ of itself in 3 years at a certain rate of simple interest. What is the rate per annum?
40. Simple interest on a certain sum at a certain annual rate of interest is $\frac{1}{9}$ of the sum. If the numbers representing rate percent and time in years be equal, then what is the rate of interest?
41. Simple interest on a certain amount is $\frac{9}{16}$ of the principal. If the numbers representing the rate of interest in percent and time in years be equal, the time, for which is the principal lent?
42. A lends Rs. 2500 to B and a certain sum to C at the same time at 7% p.a. simple interest. If after 4 years, A altogether receives Rs.1120 as interest from B and C, then what is the sum lent to C ?
43. Two equal sums of money were lent at simple interest at 11% p.a. for $3\frac{1}{2}$ years and $4\frac{1}{2}$ years respectively. If the difference in interests for two periods was Rs. 412.50, then what is the sum?
44. If the simple interest on a certain sum for 15 months at $7\frac{1}{2}$% per annum exceeds the simple interest on the same sum for 8 months at $12\frac{1}{2}$% per annum by Rs. 32.50, then what is the sum?

45. A man invests a certain sum of money at 6% p.a. simple interest and another sum at 7% p.a. simple interest. His income from interest after 2 years was Rs. 354. One – fourth of the first sum is equal to one-fifth of the second sum. The total sum invested was:
46. A borrowed some money from B at 12% p.a. S.I. for 3 years. He then added some more money to the borrowed sum and lent it to C for the same period at 14%p.a. rate of interest. If A gains Rs. 93.90 in the whole transaction, how much money did he add from his side?
47. A person borrowed Rs. 500 @ 3% per annum S.I. and Rs. 600 @ $4\frac{1}{2}$% per annum on the agreement that the whole sum will be returned only when the total interest becomes Rs. 126. The number of years, after which the borrowed sum is to be returned, is:
48. A lent Rs. 5000 to B for 2 years and Rs. 3000 to C for 4 years on simple interest at the same rate of interest and received Rs. 2200 in all from both of them as interest. What is the rate of interest per annum?
49. A sum of Rs. 725 is lent in the beginning of a year at a certain rate of interest. After 8 months, a sum of Rs. 362.50 more is lent but at the rate twice the former. At the end of the year, Rs. 33.50 is earned as interest from both the loans., what was the original rate of interest?
50. The difference between the simple interest received from two different sources on Rs. 1500 for 3 years is Rs. 13.50. The difference between their rates of interest is?
51. Peter invested an amount of Rs. 12,000 at the rate of 10 p.c.p.a. simple intereset and another amount at the rate of 20 p.c.p.a. simple interest. The total interest earned at the end of one year on the total amount invested became 14 p.c.p.a. Find the total amount invested.
52. What should be the least number of years in which the simple interest on Rs. 2600 at $6\frac{2}{3}$% will be an exact number of rupees?
53. The rates of simple interest in two banks A and B are in the ratio 5 : 4 A person wants to deposit his total savings in two banks in such a way that he received equal half-yearly interest from both. He should deposit the savings in banks A and B in the ratio:
54. A sum was put at simple interest at a certain rate for 2 years. Had it been put at 3% higher rate, it would have fetched Rs. 72 more. What is the sum?
55. If the annual rate of simple interest increases from 10% to $12\frac{1}{2}$%, a man's yearly income increases by Rs. 1250. His principal is:

56. A money lender finds that due to a fall in the annual rate of interest from 8% to $7\frac{3}{4}\%$, his yearly income diminishes by Rs. 61.50. His capital is:
57. What annual payment will discharge a debt of Rs. 6450 due in 4 years at 5% simple interest?
58. A sum of Rs. 10 is lent to be returned in 11 monthly instalments of Rs. 1 each, interest being simple. What is the rate of interest?
59. A person takes a loan of Rs. 200 at 5% simple interest. He returns Rs. 100 at the end of 1 year. In order to clear his dues at the end of 2 years, he would pay:
60. The price of a T.V. set worth Rs. 20,000 is to be paid in 20 instalments of Rs. 1000 each. If the rate of interest be 6% per annum, and the first instalment be paid at the time of purchase, then the value of the last instalment covering the interest as well will be:
61. If the rate increases by 2%, the simple interest received on a sum of money increases by Rs. 108. If the time period is increased by 2 years, the simple interest on the same sum increases by Rs.; 180. What is the sum?
62. Mr. Thomas invested an amount of rs. 13,900 divided in two different schemes A and B at the simple interest rate of 14% p.a and 11% p.a. respectively. If the total amount of simple interest earned in 2 years be rs. 3508, what was the amount invested in Scheme B?
63. A sum of Rs. 2600 is lent out in two parts in such a way that the interest on one part at 10% for 5 years is equal to that on another at 9% for 6 years. The sum lent out at 10% is:
64. A sum of Rs. 1550 was lent partly at 5% and partly 8|% p.a. simple interest. The total interest received after 3 years was Rs. 300. The ratio of the money lent at 5% to that lent at 8% is:
65. A man lends Rs. 10,000 in four parts. If he gets 8% on Rs. 2000, $7\frac{1}{2}\%$ on Rs. 4000 and $8\frac{1}{2}\%$ on Rs. 1400; what percent must he get for the remainder, if his average annual interest is 8.13%?
66. An amount of Rs. 1,00,000 is invested in two types of shares. The first yields an interest of 9% p.a. and the second, 11%p.a. If the total interest at the end of one year is $9\frac{3}{4}\%$, then the amount invested in each share was:
67. David invested certain amount in three different schemes A,B and C with the rate of interest 10% p.a., 12% p.a. and 15% p.a. respectively. If the total interest accrued in one

year was Rs. 3200 and the amount invested in Scheme C was 150% of the amount invested in Scheme A and 240% of the amount invested in Scheme B, what was the amount invested in Scheme B?

68. A person invested in all Rs. 2600 at 4%, 6% and 8% per annum simple interest. At the end of the year, he got the same interest in all the three cases. The money invested at 4% is:
69. Divide Rs. 2379 into 3 parts so that their amounts after 2,3 and 4 years respectively may be equal, the rate of interest being 5% per annum at simple interest. The first part is:
70. A man invested $\frac{1}{3}$ of his capital at 7%; $\frac{1}{4}$ at 8% and the remainder at 10%. If his annual income is Rs. 561, the capital is:

ANSWERS

1. Rs.918	2. Rs. 787.50	3. Rs. 27.30	4. Rs. 112.50
5. 4 years	6. 6%	7. $6\frac{3}{4}\%$	8. R = 6
9. Rs. 15000	10. X = 120	11. Rs. 8925	12. Rs. $\left(\frac{100}{x}\right)$
13. Rs. 252	14. Data inadequate	15. 9 years	16. Rs. 51.60
17. 8%	18. Rs. 22.50	19. Rs. 105	20. 2 : 3
21. X = 8000	22. Rs. 1200	23. Rs. 1.20	24. 10.25%
25. Rs. 698	26. Rs. 600	27. 12%	28. 4%
29. 8 years 4 months	30. $8\frac{1}{3}\%$	31. 20%	32. 18 years
33. 7 years 9 months	34. 2 alone is correct	35. Data inadequate	36. 4%
37. $1\frac{1}{4}$ years	38. 8 years	39. $5\frac{5}{9}\%$	40. $3\frac{1}{3}\%$
41. $7\frac{1}{2}$ years	42. 1500	43. X=3750	44. 3120
45. Rs. 2700	46. Rs. 105	47. 3 years	48. 10%
49. 3.46%	50. 0.3%	51. Rs. 20000	52. T=3
53. 4:5	54. X=1200	55. X=50000	56. X=24600

57. X=1500	58. $21\frac{9}{11}\%$	59. Rs. 115	60. Rs. 19000
61. Data inadequate	62. Rs. 6400	63. Rs. 1350	64. 16:15
65. R=9	66. Rs. 37500	67. Rs. 5000	68. Rs. 1200
69. Rs. 828	70. 6600		

COMPOUND INTEREST

Compound Interest: Sometimes it so happens that the borrower and the lender agree to fix up a certain unit of time, say *yearly* or *half-yearly* or *quarterly* to settle the previous account.
In such cases, the amount after first unit of time becomes the principal for the second unit, the amount after second unit becomes the principal for the third unit and so on.
After a specified period, *the difference between the amount and the money borrowed is called the* ***Compound Interest*** (*abbreviated as C.I.*) *for that period*

Let Principal = P, Rate = R% per annum, Time = *n* years.

I. When interest is compounded Annually:

Amount = $P\left(1+\frac{R}{100}\right)^n$

II. When interest is compounded Half-yearly:

Amount = $P\left(1+\frac{\left(\frac{R}{2}\right)}{100}\right)^{2n}$

III. When interest is compounded Quarterly:

Amount = $P\left(\frac{\left(\frac{R}{4}\right)}{100}\right)^{4n}$

IV. When interest is compounded Annually but time is in fraction, say $3\frac{2}{5}$ years.

Amount = $P\left(1+\frac{R_1}{100}\right)^3\left(1+\frac{\frac{2}{5}R}{100}\right)$

V. When rates are different for different years, say *R*1%, *R*2%, *R*3% for 1st, 2nd and 3rd year respectively.

Amount = $P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$

VI. Present worth of Rs. *x* due *n* years hence is given by:

Present Worth = $\frac{x}{\left(1+\frac{R}{100}\right)^n}$

SOLVED PROBLEMS

Example 1. Find compound interest on Rs. 7500 at 4% per annum for 2 years, compounded annually.
Solution.

Amount = Rs. $\left[7500\times\left(1+\frac{4}{100}\right)^2\right]$

= Rs. $\left(7500\times\frac{26}{25}\times\frac{26}{25}\right)$

= Rs.8112

∴ C.I = Rs. $(8112-7500) = Rs.612.$

Examples 2. Find the compound interest on Rs. 8000 at 15% per annum for 2 years 4 months, compounded annually.

Solution.

Time = 2 years 4 months = $2\frac{4}{12}$ years = $2\frac{1}{3}$ years.

Amount = Rs. $\left[8000 \times \left(1+\frac{15}{100}\right)^2 \times \left(1+\frac{\frac{1}{3}\times 15}{100}\right)\right]$

= Rs. $\left(8000 \times \frac{23}{20} \times \frac{23}{20} \times \frac{21}{20}\right)$

= Rs. 11109

∴ C.I = Rs. $(11109 - 8000) = Rs.3109$.

Example 3. Find the compound interest on Rs. 10,000 in 2 years at 4% per annum, the interest being compounded half yearly.

Solution.

Principal = Rs. 10000, Rate = 2% per half-year, Time = 2 years = 4 half-years.

Amount = Rs. $\left[10000 \times \left(1+\frac{2}{100}\right)^4\right]$

= Rs. $\left(10000 \times \frac{51}{50} \times \frac{51}{50} \times \frac{51}{50} \times \frac{51}{50}\right)$

= Rs. 10824.32

∴ C.I = Rs. $(10824.32 - 10000) = Rs.824.32$.

Example 4. Find the compound interest on Rs. 16,000 at 20% per annum for 9 months, compounded quarterly.

Solution.

Principal = Rs. 16,000; Time = 9 months = 3 quarters;

Rate = 20% per annum = 5% per quarter.

Amount = Rs. $\left[16000 \times \left(1+\frac{5}{100}\right)^3\right]$

= Rs. $\left(16000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}\right)$

= Rs. 18522.

∴ C.I = Rs. $(18522 - 16000) = Rs.2522$.

Example 5. If the simple interest on a sum of money at 5% per annum for 3 years is Rs. 1200, find the compound interest on the same sum for the same period at the same rate.

Solution.

Rate = 5% per annum; Time = 3 years; S.I. = Rs. 1200.

Principal = Rs. $\left(\frac{100\times 1200}{3\times 5}\right)$

= Rs. 8000.

Amount = Rs. $\left[8000 \times \left(1+\frac{5}{100}\right)^3\right]$

= Rs. $\left(16000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}\right)$

= Rs. 9261.

∴ C.I = Rs. $(9261 - 8000) = Rs.1261$.

Example 6. In what time will Rs. 1000 become Rs.1331 at 10% per annum compounded annually?

Solution.

Principal = Rs. 1000; Amount = 1331; Rate = 10% per annum

Let the time be n years. Then

$$\left[1000\left(1+\frac{10}{100}\right)^{n}\right]=1331$$
$$\Rightarrow\left(\frac{11}{10}\right)^{n}=\left(\frac{1331}{1000}\right)$$
$$\Rightarrow\left(\frac{11}{10}\right)^{n}=\left(\frac{11}{10}\right)^{3}$$
$\therefore n=3\ years$

Example 7. If Rs. 500 amounts to Rs. 583.20 in two years compounded annually, find the rate of interest per annum.
Solution.
Principal = Rs. 500; Amount = Rs.583; Time = 2 years

Let the time be R% per annum. Then,
$$\left[500\left(1+\frac{R}{100}\right)^{2}\right]=583.20$$
$$\left(1+\frac{R}{100}\right)^{2}=\frac{5832}{5000}=\frac{11664}{10000}$$
$$\therefore\quad\left(1+\frac{R}{100}\right)^{2}=\left(\frac{108}{100}\right)^{2}$$
$$1+\frac{R}{100}=\frac{108}{100}$$
$$R=8$$
So, rate = 8% per annum.

Example 8. If the compound interest on a certain sum at $16\frac{2}{3}\%$ for 3 years is Rs. 1270, find the simple interest on the same sum at the same rate and for the same period.
Solution.
Let the sum be Rs. x. Then,
$$C.I.=\left[x\times\left(\frac{50}{3\times100}\right)^{3}-x\right]=\left(\frac{343x}{216}-x\right)=\frac{127x}{216}$$
$$\therefore\frac{127x}{216}=1270$$
$$x=\frac{1270\times216}{127}=2160$$
Thus, the sum is Rs. 2160
$\therefore$ S.I.$= Rs.\left(2160\times\frac{50}{3}\times3\times\frac{1}{100}\right)=Rs.1080.$

Example 9. The difference between the compound interest and simple interest on a certain sum at 10% per annum for 2 years is Rs. 631. Find the sum.
Solution.
Let the sum be Rs. x. Then,
$$C.I=x\left(1+\frac{10}{100}\right)^{2}-x=\frac{21x}{100}$$
$$S.I.\left(\frac{X\times10\times2}{100}\right)=\frac{x}{5}$$
$$\therefore(C.I.)-(S.I.)=\left(\frac{21x}{100}-\frac{x}{5}\right)=\frac{x}{100}$$
$$\therefore\frac{x}{100}=631$$
$$x=63100$$
Hence, the sum is Rs. 63,100.

Example 10. The difference between the compound interest and the simple interest accrued on an amount of Rs. 18,000 in 2 years was Rs. 405. What was the rate of interest p.c.p.a.?

Solution.

Let the rate be R% p.a. Then,

$$\left[18000\left(1+\frac{R}{100}\right)^2-18000\right]-\left(\frac{18000\times R\times 2}{100}\right)=405$$

$$\Leftrightarrow 18000\left[\frac{(100+R)^2}{10000}-1-\frac{2R}{100}\right]=405$$

$$\Leftrightarrow 18000\left[\frac{(100+R)^2-10000-200R}{10000}\right]=405$$

$$\Leftrightarrow \frac{9}{5}R^2=405 \Leftrightarrow R^2=\left(\frac{405\times 5}{9}\right)=225$$

$$\Leftrightarrow R=15.$$

$$\therefore Rate=15\%.$$

Example 11. Divide Rs. 1301 between A and B, so that the amount of A after 7 years is equal to the amount of B after 9 years, the interest being compounded at 4% per annum.

Solution.

Let the two parts be Rs. x and Rs. (1301 -x)

$$x\left(1+\frac{4}{100}\right)^7=(1301-x)\left(1+\frac{4}{100}\right)^9$$

$$\Leftrightarrow \frac{x}{(1301-x)}=\left(1\div\frac{4}{100}\right)^2=\left(\frac{26}{25}\times\frac{26}{25}\right)$$

$$\Leftrightarrow 625x=676\,(1301-x) \Leftrightarrow 1301x=676\times 1301 \Leftrightarrow x=676$$

So , the two parts are Rs. 676 and Rs. (1301 – 676)

i.e. Rs. 676 and Rs. 625.

Example 12. A certain sum amounts to Rs. 7350 in 2 years and to Rs. 8575 in 3 years. Find the sum and rate percent.

Solution.

S.I. on Rs. 7350 for 1 year = Rs. (8575 – 7350) = Rs. 1225

$$\therefore Rate=\left(\frac{100\times 1225}{7350\times 1}\right)\%=16\frac{2}{3}\%$$

Let the sum be Rs. x. Then,

$$x\left(1+\frac{50}{3\times 100}\right)^2=7350 \Leftrightarrow x\times\frac{7}{6}\times\frac{7}{6}=7350$$

$$\Leftrightarrow x=\left(7350\times\frac{36}{49}\right)=5400.$$

$$\therefore Sum=Rs.5400$$

Example 13. A sum of money amounts to Rs. 6690 after 3 years and to Rs. 10,035 after 6 years on compound interest. Find the sum.

Solution.

Let the sum be Rs. P. Then,

$$P\left(1+\frac{R}{100}\right)^3=6690 \quad \ldots\ldots(i)$$

and $P\left(1+\frac{R}{100}\right)^6=10035 \quad \ldots\ldots(ii)$

on dividing, we get $\left(1+\frac{R}{100}\right)^3=\frac{10035}{6690}=\frac{3}{2}$.

Substituting this value in (i), we get:

$$P\times\frac{3}{2}=6690\ or\ P=\left(6690\times\frac{2}{3}\right)=4460.$$

Hence, the sum is Rs. 4460.

Example 14. A sum of money doubles itself at compound interest in 15 years. In how many years will it become eight times?

Solution.

$$P\left(1+\frac{R}{100}\right)^{15}=2P \implies \left(1+\frac{R}{100}\right)^{15}=\frac{2P}{P}=2 \qquad \ldots\ldots(i)$$

Let $P\left(1+\frac{R}{100}\right)^{n}=8P \implies \left(1+\frac{R}{100}\right)^{n}=8=2^3=\left\{\left(1+\frac{R}{100}\right)^{15}\right\}^3$ [using (i)]

$$\implies \left(1+\frac{R}{100}\right)^{n}=\left(1+\frac{R}{100}\right)^{45} \implies n=45.$$

Thus, the required time = 45 years.

Example 15. What annual payment will discharge a debt of Rs. 7620 due in 3 years at $16\frac{2}{3}\%$ per annum compound interest?

Solution.

Let each instalment be Rs. x. Then,

(P.W. of Rs. x due 1 year hence) + (P.W. of Rs. x due 2 years hence) + (P.W. of Rs. x due 3 years hence) = 7620.

$$\therefore \frac{x}{\left(1+\frac{50}{3\times100}\right)}+\frac{x}{\left(1+\frac{50}{3\times100}\right)^2}+\frac{x}{\left(1+\frac{50}{3\times100}\right)^3}=7620$$

$$\Leftrightarrow \frac{6x}{7}+\frac{36x}{49}+\frac{216x}{343}=7620 \quad \Leftrightarrow 294x+252x+216x=7620\times343$$

$$\Leftrightarrow x=\left(\frac{7620\times343}{762}\right)=3430.$$

$\therefore$ Amount of each instalment = Rs. 3430.

Example 16. Albert invested an amount of Rs. 8000 in a fixed deposit scheme for 2 years at compound interest rate 5 p.c.p.a. How much amount will Albert get on maturity of the fixed deposit?

Solution.

$$\text{Amount} = Rs.\left[8000\times\left(1+\frac{5}{100}\right)^2\right]$$

$$= Rs.\left(8000\times\frac{21}{20}\times\frac{21}{20}\right)$$

$$= Rs.\,8820.$$

Example 17. A man saves Rs. 200 at the end of each year and lends the money at 5% compound interest. How much will it become at the end of 3 years?

Solution.

$$\text{Amount} = Rs.\left[200\left(1+\frac{5}{100}\right)^3+200\left(1+\frac{5}{100}\right)^2+200\left(1+\frac{5}{100}\right)\right]$$

$$= Rs.\left[200\times\frac{21}{20}\times\frac{21}{20}\times\frac{21}{20}+200+\frac{21}{20}\times\frac{21}{20}+200\times\frac{21}{20}\right]$$

$$= Rs.\left[200\times\frac{21}{20}\left(\frac{21}{20}\times\frac{21}{20}+\frac{21}{20}+1\right)\right]=Rs.\,662.02.$$

Example 18. A bank offers 5% compound interest calculated on half-yearly basis. A customer deposits Rs. 1600 each on 1st January and 1st July of a year. At the end of the year what will be the amount he would have gained by way of interest.

Solution.

$$\text{Amount} = Rs.\left[1600 \times \left(1 + \frac{5}{2\times100}\right)^2 + 1600 \times \left(1 + \frac{5}{2\times100}\right)\right]$$
$$= Rs.\left[1600 \times \frac{41}{40} \times \frac{41}{40} + 1600 \times \frac{41}{40}\right]$$
$$= Rs.\left[1600 \times \frac{41}{40}\left(\frac{41}{40} + 1\right)\right]$$
$$= Rs.\left(\frac{1600\times41\times81}{40\times40}\right) = Rs.3321.$$
$$\therefore C.I. = Rs.(3321 - 3200) = Rs.121.$$

Example 19. Find the compound interest on Rs. 15,625 for 9 months at 16% per annum compounded quarterly.

Solution.

$$P = Rs.15625,\ \ n = 9\ months = 3\ quarters, R = 16\%\ p.a. = 4\%\ per\ quarter.$$
$$\text{Amount} = Rs.\left[15625 \times \left(1 + \frac{4}{100}\right)^3\right] = Rs.\left(15625 \times \frac{26}{25} \times \frac{26}{25} \times \frac{26}{25}\right) = Rs.17576$$
$$\therefore C.I. = Rs.(17576 - 15625) = Rs.1951.$$

Example 20. The compound interest on Rs. 30,000 at 7% per annum is Rs. 4347. Find the period (in years)?

Solution.

$$\text{Amount} = Rs.(30000 + 4347) = Rs.34347.$$
Let the time be n years. Then,
$$30000\left(1 + \frac{7}{100}\right)^n = 34347 \iff \left(\frac{107}{100}\right)^7 = \frac{34347}{30000} = \frac{11449}{10000} = \left(\frac{107}{100}\right)^2$$
$$\therefore n = 2\ years.$$

Example 21. In how many years will a sum of Rs. 800 at 10% per annum compounded semi-annually become Rs.926.10?

Solution.

Let the time be n years. Then,
$$800 \times \left(1 + \frac{5}{100}\right)^{2n} = 926.10 \quad or\ \left(1 + \frac{5}{100}\right)^{2n} = \frac{9261}{8000}$$
$$or\ \left(\frac{21}{20}\right)^{2n} = \left(\frac{21}{20}\right)^3\ or\ \ 2n = 3\ or\ n = \frac{3}{2}.$$
$$\therefore n = 1\frac{1}{2}\ years.$$

Example 22. If the compound interest on a sum for 2 years at $12\frac{1}{2}\%$ per annum is Rs. 510, then find the simple interest on the same sum at the same rate for the same period of time.

Solution.

Let the sum be Rs. P. Then,
$$\left[P\left(1 + \frac{25}{2\times100}\right)^2 - P\right] = 510 \quad or \quad P\left[\left(\frac{9}{8}\right)^2 - 1\right] = 510\ or\ P = \left(\frac{510\times64}{17}\right) = 1920.$$
$$\therefore Sum = Rs.1920.$$
$$\text{So, S.I.} = Rs.\left(\frac{1920\times25\times2}{2\times100}\right) = Rs.480.$$

Example 23. The difference between compound interest and simple interest on an amount Rs. 15,000 for 2 years is Rs. 96. What is the rate of interest per annum?

Solution.

$$\left[15000 \times \left(1 + \frac{R}{100}\right)^2 - 15000\right] - \left(\frac{15000\times R\times2}{100}\right) = 96$$

$\Leftrightarrow 15000\left[\left(1+\frac{R}{100}\right)^2-1-\frac{2R}{100}\right]=96$

$\Leftrightarrow 15000\left[\frac{(100+R)^2-10000-200R}{10000}\right]=96$

$\Leftrightarrow R^2=\frac{96\times2}{3}=64 \quad \Leftrightarrow R=8.$

$\therefore Rate=8\%$

Example 24. A sum of money lent at compound interest for 2 years at 20% per annum would fetch Rs. 482 more, if the interest was payable half – yearly than if it was payable annually. Find the sum.

Solution.

Let the sum be Rs. x. then,

C.I. when compounded half-yearly $=\left[x\times\left(1+\frac{10}{100}\right)^4-x\right]=\frac{4641}{10000}x.$

C.I. when compounded annually $=\left[x\times\left(1+\frac{20}{100}\right)^2-x\right]=\frac{11}{25}x.$

$\therefore \frac{4641}{10000}x-\frac{11}{25}x=482 \; or \; x=\frac{482\times10000}{241}=20000.$

Example 25. A person lent out a certain sum on simple interest and the same sum on compound interest at a certain rate of interest per annum. He noticed that the ration between the difference of compound interest and simple interest of 3 years and that of 2 years is 25 : 8. Find the rate of interest per annum.

Solution.

Let the principal be Rs. P and rate of interest be R% per annum.

Difference of C.I. and S.I. for 2 years

$=\left[P\times\left(1+\frac{R}{100}\right)^2-P\right]-\left(\frac{P\times R\times2}{100}\right)=\frac{PR^2}{104}$

Difference of C.I. and S.I. for 3 years

$=\left[P\times\left(1+\frac{R}{100}\right)^3-P\right]-\left(\frac{P\times R\times3}{100}\right)=\frac{PR^2}{104}\left(\frac{300+R}{100}\right)$

$\therefore \frac{\frac{PR^2}{104}\left(\frac{300+R}{100}\right)}{\frac{PR^2}{104}}=\frac{25}{8} \quad \Rightarrow\left(\frac{300+R}{100}\right)=\frac{25}{8} \quad \Longrightarrow R=\frac{100}{8}=12\frac{1}{2}\%.$

Example 26. A sum of Rs. 12,000 deposited at compound interest becomes double after 5 years. After 20 years, what will be the sum.

Solution.

$12000\times\left(1+\frac{R}{100}\right)^5=24000 \;\Rightarrow\left(1+\frac{R}{100}\right)^5=2$

$\therefore\left[\left(1+\frac{R}{100}\right)^5\right]^4=2^4=16 \;\Rightarrow\left(1+\frac{R}{100}\right)^{20}=16 \qquad \Rightarrow P\left(1+\frac{R}{100}\right)^{20}=16P$

$\Rightarrow 12000\left(1+\frac{R}{100}\right)^{20}=16\times12000=192000$

Example 27. A sum of money placed at compound interest doubles itself in 5 years. How many years it will amount to eight times itself at the same rate of interest?

Solution.

$P\left(1+\frac{R}{100}\right)^5=2P \qquad \Rightarrow\left(1+\frac{R}{100}\right)^5=2 \qquad$ …….. (i)

Let $P\left(1+\frac{R}{100}\right)^n=8P \quad \Rightarrow\left(1+\frac{R}{100}\right)^n=8=2^3=\left\{\left(1+\frac{R}{100}\right)^5\right\}^3$ [using (i)]

$$\Rightarrow \left(1+\frac{R}{100}\right)^{n} = \left(1+\frac{R}{100}\right)^{15} \Rightarrow n = 15.$$

$\therefore$ Required time = 15 years.

Example 28. What annual payment will discharge a debt of Rs. 1025 due in 2 years at the rate of 5% compound interest?

Solution.

Let each instalment be Rs. x. Then,

$$\frac{x}{\left(1+\frac{5}{100}\right)}+\frac{x}{\left(1+\frac{5}{100}\right)^{2}} = 1025 \quad \Leftrightarrow \frac{20x}{21}+\frac{400x}{441} = 1025$$

$$\Rightarrow 820x = 1025 \times 441 \qquad \Leftrightarrow x = \left(\frac{1025\times441}{820}\right) = 551.25.$$

So, value of each instalment = Rs. 551.25.

EXERCISE

1. What will be the compound interest on a sum of Rs. 25,000 after 3 years at the rate of 12.p.c.p.a?
2. The compound interest on Rs. 20,480 at $6\frac{1}{4}\%$ per annum for 2 years 73 days, is:
3. Sam invested Rs. 15,000 @ 10% per annum for one year. If the interest is compounded half-yearly, then the amount received by Sam at the end of the year will be;
4. What is the difference between the compound interests on Rs. 5000 for $1\frac{1}{2}$ years at 4% per annum compounded yearly and half-yearly?
5. If the simple interest on a sum of money for 2 years at 5% per annum is Rs. 50, what is the compound interest on the same sum at the same rate and for the same time?
6. What will be the difference between simple and compound interest @ 10% per annum on a sum of Rs. 1000 after 4 years?
7. The difference between simple interest and compound interest on Rs. 1200 for one year at 10% per annum reckoned half-yearly is:
8. At what rate of compound interest per annum will a sum of Rs. 1200 become Rs. 1348.32 in 2 years?
9. The principal that amounts to Rs. 4913 in 3 years at $6\frac{1}{4}\%$ per annum compound interest compounded annually, is:
10. The present worth of Rs. 169 due in 2 years at 4% per annum compound interest is:
11. The compound interest on a certain sum for 2 years at 10% per annum is Rs. 525. The simple interest on the same sum for double the time at half the rate percent per annum is:
12. The simple interest on a certain sum of money for 3 years at 8% per annum is half the compound interest on Rs. 4000 for 2 years at 10% per annum. The sum placed on simple interest is:
13. There is 60% increase in an amount in 6 years at simple interest. What will be the compound interest of Rs. 12,000 after 3 years at the same rate?
14. The difference between simple and compound interest compounded annually on certain sum of money for 2 years at 4% per annum is Re.1. The sum (in Rs.) is:
15. The compound interest on a sum of money for 2 years is Rs. 832 and the simple interest on the same sum for the same period is Rs. 800. The difference between the compound interest and the simple interest for 3 years will be:

16. The difference between the simple interest on a certain sum at the rate of 10% per annum for 2 years and compound interest which is compounded every 6 months is Rs. 124.05. what is the principal sum?
17. The difference between compound interest and simple interest on a sum for 2 years at 10% per annum, when the interest is compounded annually is Rs. 16. If the interest were compounded half-yearly, the difference in two interest would be:
18. On a sum of money, the simple interest for 2 years is Rs. 660, while the compound interest is Rs. 696.30, the rate of interest being the same in both the cases. The rate of interest is:
19. The effective annual rate of interest corresponding to a nominal rate of 6% per annum payable half – yearly is:
20. Mr. Dua invested money in two schemes A and B offering compound interest @ 8 p.c.p.a. and 9 p.c.p.a respectively. If the total amount of interest accrued through two schemes together in two years was Rs. 4818.30 and the total amount invested was Rs. 27,000, what was the amount invested in Scheme A?
21. A sum of money invested at compound interest amounts to Rs. 800 in 3 years and to Rs. 840 in 4 years. The rate of interest per annum is:
22. A sum of money invested at compound interest amounts to Rs. 4624 in 2 years and to Rs. 4913 in 3 years. The sum of money is:
23. A sum of money becomes Rs.13,380 after 3 years and Rs. 20,070 after 6 years on compound interest. The sum is:
24. If a sum on compound interest becomes three times in 4 years, then with the same interest rate, the sum will become 27 times in:
25. The least number of complete years in which a sum of money put out at 20% compound interest will be more than doubled is:
26. A man borrows Rs. 2550 to be paid back with compound interest at the rate of 4% per annum by the end of 2 years in two equal yearly instalments. How much will each instalment be?
27. A man borrows Rs. 12,500 at 20% compound interest. At the end of every year he pays Rs. 2000 as part repayment. How much does he still owe after three such instalments?
28. A sum of money is borrowed and paid back in two annual instalments of Rs. 882 each allowing 5% compound interest. The sum borrowed was:

ANSWERS

1. Rs. 10123.20	2. Rs. 2929	3. Rs. 16537.50	4. Rs. 2.04
5. Rs. 51.25	6. Rs. 64.10	7. Rs. 123	8. R=6%
9. Rs. 4096	10. Rs. 156.25	11. Rs. 500	12. Rs. 1750
13. Rs. 3972	14. Rate = 8%	15. x = 625	16. Rs. 98.56
17. P = 8000	18. Rs. 24.81	19. 11%	20. Rs. 106.09
21. x = 12000	22. 5%	23. Rs. 4096	24. 12 years
25. 4 years	26. x = 1352	27. Rs. 14320	28. Rs. 1640

UNIT II
TIME AND WORK

INTRODUCTION

1. If A can do a piece of work in n days, then A's 1 day's work = $\frac{1}{n}$.
2. If A's 1 day's work $= \frac{1}{n}$, then A can finish the work in n days.
3. If A is thrice as good a workman as B, then;
   Ratio of work done by A and B = 3 : 1.
   Ratio of times taken by A and B to finish a work = 1 : 3.

SOLVED PROBLEMS

Example 1. Worker A takes 8 hours to do job. Worker B takes 10 hours to do the same job. How long should it take both A and , working together but independently, to do the same job?
Solution.

A's 1 hour's work = $\frac{1}{8}$,, B's 1 hour's work = $\frac{1}{10}$.
(A+B)'s 1 hour's work = $\left(\frac{1}{8}+\frac{1}{10}\right)=\frac{9}{40}$.
Therefore, Both A and B will finish the work in $\frac{40}{9}=4\frac{4}{9}$ days.

Example 2. A and B together can complete a piece of work in 4 days. If A alone can complete the same work in 12 days, in how many days can B alone complete that work?
Solution.

$(A+B)'s\ 1\ days'swork=\frac{1}{4}, A's1\ day's\ work\ =\frac{1}{12}.$
$\therefore B's\ 1\ day's\ work\ =\left(\frac{1}{4}-\frac{1}{12}\right)=\frac{1}{6}.$
Hence, B alone can complete the work in 6 days.

Example 3. A can do a piece of work in 7 days of 9 hours each and B can do it in 6 days of 7 hours each. How long will they take to do it, working together $8\frac{2}{5}$ hours a day?
Solution.

A can complete the work in $(7\times 9)=63\ hours.$
B can complete the work in $(6\times 7)=42\ hours.$
∴ A's 1 hour's work = $\frac{1}{63}$ and B's 1 hour's work = $\frac{1}{42}$.
$(A+B)'s$ 1 hour's work = $\left(\frac{1}{63}+\frac{1}{42}\right)=\frac{5}{126}$.
Both will finish the work in $\left(\frac{126}{5}\right)hrs.$
Number of days of $8\frac{2}{5}$ hrs each = $\left(\frac{126}{5}\times\frac{5}{42}\right)=3\ days.$

Example 4. A and B can do a piece of work in 18 days; B and C can do it in 24 days; A and C can do it in 36 days. In how many days will A,B and C finish it, working together and separately?
Solution.

$(A+B)'s$ 1 day's work = $\frac{1}{18}$, $(B+C)'s$ 1 day's work = $\frac{1}{24}$.

and $(A+C)'s$ 1 day's work $= \frac{1}{36}$.

Adding, we get : $2(A+B+C)'s$ 1 day's work $= \left(\frac{1}{18}+\frac{1}{24}+\frac{1}{36}\right) = \frac{9}{72} = \frac{1}{8}$.

$\therefore (A+B+C)'s$ 1 day's work $= \frac{1}{16}$.

Thus, A, B, and C together can finish the work in 16 days.

Now, A's 1 day's work= $[(A+B+C)'s\ 1\ day's\ work] - [(B+C)'s\ 1\ day's\ work]$

$$= \left(\frac{1}{16}-\frac{1}{24}\right) = \frac{1}{48}.$$

$\therefore$ A alone can finish the work in 48 days.

Similarly, B's 1 day's work $= \left(\frac{1}{16}-\frac{1}{36}\right) = \frac{5}{144}$.

$\therefore B$ alone can finish the work in $\frac{144}{5} = 28\frac{4}{5}$ days.

And, C's 1 day's work $= \left(\frac{1}{16}-\frac{1}{18}\right) = \frac{1}{144}$.

$\therefore$ C alone can finish the work in 144 days.

Example 5. A is twice as good a workman as B and together they finish a piece of work in 18 days. In how many days will A alone finish the work?

Solution.

$(A's\ 1\ day's\ work) : (B's\ 1\ day's\ work) = 2 : 1.$

$(A+B)'s\ \ 1\ day's\ work\ = \frac{1}{18}$.

Divide $\frac{1}{18}$ in the ration 2 : 1

$\therefore$ A's 1 day's work $= \left(\frac{1}{18}\times\frac{2}{3}\right) = \frac{1}{27}$.

Hence, A alone can finish the work in 27 days.

Example 6. A can do a certain job in 12 days. B is 60% more efficient that A. How many days does B alone take to do the same job?

Solution.

Ratio of time taken by A and B $= 160 : 100 = 8 : 5$.

Suppose B alone takes x days to do the job.

Then, $8 : 5 :: 12 : x \ \Rightarrow 8x = 5\times 12 \ \Rightarrow x = 7\frac{1}{2}\ days.$

Example 7. A can do a piece of work in 80 days. He works at it for 10 days and then B alone finishes the remaining work in 42 days. In how much time will A and B, working together, finish the work?

Solution.

Work done by A in 10 days $= \left(\frac{1}{80}\times 10\right) = \frac{1}{8}$.

Remaining work $= \left(1-\frac{1}{8}\right) = \frac{7}{8}$.

Now, $\frac{7}{8}$ work is done by B in 42 days.

Whole work will be done by B in $\left(42\times\frac{8}{7}\right) = 48\ days.$

$\therefore$ A's 1 day's work $= \frac{1}{80}$ and B's 1 day's work $= \frac{1}{48}$.

$\therefore (A+B)'s$ 1 day's work $= \left(\frac{1}{80}+\frac{1}{48}\right) = \frac{8}{240} = \frac{1}{30}$.

Hence, both will finish the work in 30 days.

Example 8. A and B undertake to do a piece of work for Rs. 600. A alone can do it in 6 days while B alone can do it in 8 days. With the help of C, they finish it in 3 days. Find the share of each.

Solution.

C's 1 day's work $= \frac{1}{3} - \left(\frac{1}{6} + \frac{1}{8}\right) = \frac{1}{24}$.

$\therefore A : B : C =$ Ratio of their 1 day's work $= \frac{1}{6} : \frac{1}{8} : \frac{1}{24} = 4 : 3 : 1.$

$\therefore$ A's share $= Rs.\left(600 \times \frac{4}{8}\right) = Rs.300$, B's share $= Rs.\left(600 \times \frac{3}{8}\right) = Rs.225.$

C's share $= Rs.[600 - (300 + 225)] = Rs.75.$

Example 9. A and B working separately can do a piece of work in 9 and 12 days respectively. If they work for a day alternately, A beginning, in how many days, the work will be completed?

Solution.

(A+B)'s 2 day's work $= \left(\frac{1}{9} + \frac{1}{12}\right) = \frac{7}{36}$.

Work done in 5 pairs of days $= \left(5 \times \frac{7}{36}\right) = \frac{35}{36}$.

Remaining work $= \left(1 - \frac{35}{36}\right) = \frac{1}{36}$.

On 11th day, it is A's turn. $\frac{1}{9}$ work is done by him in 1 day.

$\frac{1}{36}$ work is done by him in $\left(9 \times \frac{1}{36}\right) = \frac{1}{4} day.$

$\therefore$ Total time taken $= \left(10 + \frac{1}{4}\right)$ days $= 10\frac{1}{4}$ days.

Example 10. 45 men can complete a work in 16 days. Six days after they started working, 30 more men joined them. How many days will they now take to complete the remaining work?

Solution.

(45×16) men can complete the work in 1 day.

$\therefore$ 1 man's 1 day's work $= \frac{1}{720}$

45 men's 6 day's work $= \left(\frac{1}{16} \times 6\right) = \frac{3}{8}$.

Remaining work $= \left(1 - \frac{3}{8}\right) = \frac{5}{8}$.

75 men's 1 day's work $= \frac{75}{720} = \frac{5}{48}$.

Now, $\frac{5}{48}$ work is done by them in 1 day.

$\therefore \frac{5}{8}$ work is done by them in $\left(\frac{48}{5} \times \frac{5}{8}\right) = 6\ days.$

Example 11. 2 men and 3 boys can do a piece of work in 10 days while 3 men and 2 boys can dop the same work in 8 days. In how many days can 2 men and 1 boy do the work?

Solution.

Let 1 man's 1 day's work = x and 1 boy's 1 day's work = y.

Then, $2x + 3y = \frac{1}{10}\ and\ 3x + 2y = \frac{1}{8}$.

Solving, we get : $x = \frac{7}{200}\ \ and\ y = \frac{1}{100}$.

$\therefore (2\ men + 1\ boy)'s\ \ 1\ day's\ work = \left(2 \times \frac{7}{200} + 1 \times \frac{1}{100}\right) = \frac{16}{200} = \frac{2}{25}$.

So, 2 men and 1 boy together can finish the work in $\frac{25}{2} = 12\frac{1}{2}$ days.

Example 12. A does a work in 10 days and B does the same work in 15 days. In how many days they together will do the same work?

Solution.

A's 1 day's work $= \frac{1}{10}$ and B's 1 day's work $= \frac{1}{15}$.

$\therefore (A+B)'s$ 1 day's work $= \left(\frac{1}{10}+\frac{1}{15}\right) = \frac{1}{6}$.

So, both together will finish the work in 6 days.

Example 13. A, B and C can complete a piece of work in 24, 6 and 12 days respectively. Working together, they will complete the same work in:

Solution.

$(A+B+C)'s$ 1 day's work $= \left(\frac{1}{24}+\frac{1}{6}+\frac{1}{12}\right) = \frac{7}{24}$.

So A, B and C together will complete the job in $\frac{24}{7} = 3\frac{3}{7} days$.

Example 14. A man can do a job in 15 days. His father takes 20 days and his son finishes it in 25 days. How long will they take to complete the job if they all work together?

Solution.

1 day's work of three persons $= \left(\frac{1}{15}+\frac{1}{20}+\frac{1}{25}\right) = \frac{47}{300}$.

So, all the three together will complete the work in $\frac{300}{47} \simeq 6.4\ days$.

Example 15. A take twice as much time as B or thrice as much time to finish a piece of work. Working together, they can finish the work in 2 days. B can do the work alone in:

Solution.

Suppose A, B and C take $x, \frac{x}{2}\ and \frac{x}{3}$ hours respectively to finish the work.

Then, $\left(\frac{1}{x}+\frac{2}{x}+\frac{3}{x}\right) = \frac{1}{2} \Rightarrow \frac{6}{x} = \frac{1}{2} \Rightarrow x = 12.$

So, B takes 6 hours to finish the work.

Example 16. P, Q and R are three typists who working simultaneously can type 216 pages in 4 hours. In one hour, R can type as many pages more than Q as Q can type more than P. During a period of five hours, R can type as many pages as P can during seven hours. How many pages does each of them type per hour?

Solution.

Let the number of pages typed in one hour by P,Q and R be x,y and z respectively. Then,

$x+y+z = \frac{216}{4} \quad \Rightarrow x+y+z = 54 \quad \ldots\ldots(i)$

$z-y = y-x \quad \Rightarrow 2y = x+z \quad \ldots\ldots(ii)$

$5z = 7x \quad \Rightarrow x = \frac{5}{7}z \quad \ldots\ldots(iii)$

Solving (i), (ii) and (iii), we get $x = 15, y = 18, z = 21.$

Example 17. Two workers A and B are engaged to do a work. A working alone takes 8 hours more to complete the job than if both worked together. If B worked alone, he would need $4\frac{1}{2}$ hours more to complete the job than they both working together. What time would they take to do the work together?

Solution.

Let A and B together take x hours to complete the work. Then,

A alone takes $(x+8)$ hrs and B alone takes $\left(x+\frac{9}{2}\right)$hrs to complete the work. Then,

$\frac{1}{(x+8)}+\frac{1}{\left(x+\frac{9}{2}\right)}=\frac{1}{x} \Rightarrow \frac{1}{(x+8)}+\frac{2}{(2x+9)}=\frac{1}{x} \quad \Rightarrow x(4x+25)=(x+8)(2x+9)$

$\Rightarrow 2x^2=72 \Rightarrow x^2=36$

$\Rightarrow x=6.$

Example 18. P can complete a work in 12 days working 8 hours a day. Q can complete the same work in 8 days working 10 hours a day. If both P and Q work together, working 8 hours a day, in how many days can they complete the work?

Solution.

P can complete the work in $(12\times 8)hrs.=96\ hrs.$

Q can compete the work in $(8\times 10)hrs.=80\ hrs.$

∴P's 1 hour's work $=\frac{1}{96}$ and Q's 1 hour's work $=\frac{1}{80}$.

$(P+Q)'s$ 1 hour's work $=\left(\frac{1}{96}+\frac{1}{80}\right)=\frac{11}{480}$.

So, both P and Q will finish the work in $\left(\frac{480}{11}\right)hrs.$

∴ number of days of 8 hours each $=\left(\frac{480}{11}\times\frac{1}{8}\right)=\frac{60}{11}days=5\frac{5}{11}days.$

Example 19. A and B can do a piece of work in 72 days, B and C can do it in 120 days; A and C can do it in 90 days. In what time can A alone do it?

Solution.

(A+B)'s 1 day's work $=\frac{1}{72}$;

(B+C)'s 1 day's work $=\frac{1}{120}$;

(A+C)'s 1 day's work $=\frac{1}{90}$.

Adding, we get : 2(A+B+C)'s 1 day's work $=\left(\frac{1}{72}+\frac{1}{120}+\frac{1}{90}\right)=\frac{12}{360}=\frac{1}{30}$.

$\Rightarrow (A+B+C)'s$ 1 day's work $=\frac{1}{60}$.

So, A's 1 day's work $=\left(\frac{1}{60}-\frac{1}{120}\right)=\frac{1}{120}$.

∴ A alone can do the work in 120 days.

Example 20. A can do a certain work in the same time in which B and C together can do it. If A and B together could do it in 10 days and C alone in 50 days, then B alone could do it in:

Solution.

(A+B)'s 1 day's work $=\frac{1}{10}$; C's 1 day's work $=\frac{1}{50}$.

(A+B+C)'s 1 day's work $=\left(\frac{1}{10}+\frac{1}{50}\right)=\frac{6}{50}=\frac{3}{25}$ ... ... (i)

Also, A's 1 day's work $=(B+C)'s$ 1 day's work ……(ii)

From (i) and (ii), we get : $2\times(A's\ 1\ day's\ work)=\frac{3}{25}$.

$\Rightarrow A's\ 1\ day's\ work\ =\frac{3}{50}$.

$\therefore B's\ 1\ day's\ work=\left(\frac{1}{10}-\frac{3}{50}\right)=\frac{2}{50}=\frac{1}{25}$.

So, B alone could do the work in 25 days.

Example 21. A twice as good a workman as B and together they finish a piece of work in 14 days. Find the number of days taken by A alone to finish the work

Solution.

$(A's\ 1\ day's\ work) : (B's\ 1\ day's\ work) = 2 : 1.$

$(A + B)'s\ \ 1\ day's\ work = \frac{1}{14}.$

Divide $\frac{1}{14}\ in\ the\ ratio\ 2 : 1$

$\therefore A's\ 1\ day's\ work = \left(\frac{1}{14} \times \frac{2}{3}\right) = \frac{1}{21}.$

Hence, A alone can finish the work in 21 days.

Example 22. A is thrice as good a workman as B and therefore is able to finish a job in 60 days less than B. Working together, they can do it in:

Solution.

Ratio of times taken by A and B $= 1 : 3.$

If difference of time is 2 days, B takes 3 days.

If difference of time is 60 days, B takes $\left(\frac{3}{2} \times 60\right) = 90\ days.$

So, A takes 30 days to do the work.

A's 1 day's work $= \frac{1}{30};\ \ B's\ 1\ day's\ work = \frac{1}{90}.$

(A+B)'s 1 day's work $= \left(\frac{1}{30} + \frac{1}{90}\right) = \frac{4}{90} = \frac{2}{45}.$

$\therefore$ A and B together can do the work in $\frac{45}{2} = 22\frac{1}{2}$ days.

Example 23. A is 30% more efficient than B. How much time will they, working together, take to complete a job which A alone could have done in 23 days?

Solution.

Ratio of times taken by A and B $= 100 : 130 = 10 : 13.$

Suppose B takes x days to do the work.

Then, $10 : 13 :: 23 : x \qquad \Rightarrow x = \left(\frac{23 \times 10}{10}\right) \qquad \Rightarrow x = \frac{299}{10}.$

A's 1 day's work $= \frac{1}{23};$ B's 1 day's work $= \frac{10}{229}.$

(A+B)'s 1 day's work $= \left(\frac{1}{23} + \frac{10}{299}\right) = \frac{23}{299} = \frac{1}{13}.$

$\therefore$ A and B together can complete the job in 13 days.

Example 24. A is 50% as efficient as B, C does half of the work done by A and B together. If C alone does the work in 40 days, then A, B and C together can do the work in.

Solution.

$(A's\ 1\ day's\ work) : (B's\ 1\ day's\ \ work) = 150 : 100 = 3 : 2$

Let A's and B's 1 day's work be 3x and 2x respectively.

Then, C's 1 day's work $= \left(\frac{3x + 2x}{2}\right) = \frac{5x}{2}.$

$\therefore \frac{5x}{2} = \frac{1}{40}\ or\ x = \left(\frac{1}{40} \times \frac{2}{5}\right) = \frac{1}{100}.$

A's 1 day's work $= \frac{3}{100}$; B's 1 day's work $= \frac{1}{50};$ C's 1 day's work $= \frac{1}{40}.$

$(A + B + C)'s\ 1\ day's\ work = \left(\frac{3}{100} + \frac{1}{50} + \frac{1}{40}\right) = \frac{15}{200} = \frac{3}{40}.$

So, A, B and C together can do the work in $\frac{40}{3} = 13\frac{1}{3}$ days.

Example 25. Two workers A and B working together completed a job in 5 days. If A worked twice as efficiently as he actually did and B worked $\frac{1}{3}$ as efficiently as he actually did, the work would have been completed in 3 days. A alone could complete the work in:

Solution.

Let A's 1 day's work = x and B's 1 day's work = y.

Then, $x + y = \frac{1}{5}\ and\ 2x + \frac{1}{3}y = \frac{1}{3}$.

Solving, we get : $x = \frac{4}{25}\ and\ y = \frac{1}{25}$.

∴ A's 1 day's work $= \frac{4}{25}$.

So, A alone could complete the work in $\frac{25}{4} = 6\frac{1}{4} days$.

Example 26. A and B complete a work in 15 days and 10 days respectively. They started doing the work together but after 2 days B had to leave and A alone completed the remaining work. The whole work was completed in :

Solution.

$(A + B)'s$ 1 day's work $= \left(\frac{1}{15} + \frac{1}{10}\right) = \frac{1}{6}$.

Work done by A and B in 2 days $= \left(\frac{1}{6} \times 2\right) = \frac{1}{3}$

Remaining work $= \left(1 - \frac{1}{3}\right) = \frac{2}{3}$.

Now, $\frac{1}{15}$ work is done by A in 1 day.

∴ $\frac{2}{3}$ work will be done by A in $\left(15 \times \frac{2}{3}\right) = 10\ days$.

Hence, total time taken $= (10 + 2) = 12\ years$.

Example 27. A machine P can print one lakh books in 8 hours, machine Q can print the same number of books in 10 hours while machine R can print them in 12 hours. All the machines are started at 9 a.m. while machine P is closed at 11 a.m. and the remaining two machines complete the work. Approximately at what time will the work be finished?

Solution.

$(P + Q + R)'s$ 1 hour's work $= \left(\frac{1}{8} + \frac{1}{10} + \frac{1}{12}\right) = \frac{37}{120}$.

Work done by P,Q and R in 2 hours $= \left(\frac{37}{120} \times 2\right) = \frac{37}{60}$.

Remaining work $= \left(1 - \frac{37}{60}\right) = \frac{23}{60}$.

$(Q + R)'s$ 1 hour's work $= \left(\frac{1}{10} + \frac{1}{12}\right) = \frac{11}{60}$.

Now, $\frac{11}{60}$ work is done by Q and R in 1 hour.

So, $\frac{23}{60}$ work will be done by Q and R in $\left(\frac{60}{11} \times \frac{23}{60}\right) = \frac{23}{11} hours\ \approx 2\ hours$.

So, the work will be finished approximately 2 hours after 11 a.m., i.e., around 1 p.m.

Example 28. A and B can do a piece of work in 30 days, while B and C can do the same work in 24 days and C and A in 20 days. They all work together for 10 days when B and C leave. How many days more will a take to finish the work?

Solution.

$2(A + B + C)'s\ 1\ day's\ work = \left(\frac{1}{30} + \frac{1}{24} + \frac{1}{20}\right) = \frac{15}{120} = \frac{1}{8}$

$\Rightarrow (A + B + C)'s\ 1\ day's\ work = \frac{1}{16}$.

Work done by A, B and C in 10 days $= \frac{10}{16} = \frac{5}{8}$

Remaining work $= \left(1 - \frac{5}{8}\right) = \frac{3}{8}$.

A's 1 day's work $= \left(\frac{1}{16} - \frac{1}{24}\right) = \frac{1}{48}$.

Now, $\frac{1}{48}$ work is done by A in 1day.

So, $\frac{3}{8}$ work will be done by A in $\left(48 \times \frac{3}{8}\right) = 18\ days$.

Example 29. A and B can together finish a work in 30 days. They worked together for 20 days and then B left. After another 20 days, A finished the remaining work. In how many days A alone can finish the job?

Solution.

(A+B)'s 20 day's work $\left(\frac{1}{30} \times 20\right) = \frac{2}{3}$

Remaining work $= \left(1 - \frac{2}{3}\right) = \frac{1}{3}$.

Now, $\frac{1}{3}$ work is done by A in 20 days.

Whole work will be done by A in $(20 \times 3) = 60\ days$.

Example 30. A does $\frac{4}{5}$ of a work in 20 days. He then calls in B and they together finish the remaining work in 3 days. How long B alone would take to do the whole work?

Solution.

Whole work is done by A in $\left(20 \times \frac{5}{4}\right) = 25\ days$.

Now, $\left(1 - \frac{4}{5}\right)\ i.e., \frac{1}{5}$ work is done by A and B in 3 days.

Whole work will be done by A and B in $(3 \times 5) = 15$ days.

A's 1 day's work $= \frac{1}{25}$, (A+B)'s 1 day's work $= \frac{1}{15}$.

$\therefore B's$ 1 day's work $= \left(\frac{1}{15} - \frac{1}{25}\right) = \frac{4}{150} = \frac{2}{75}$.

So, B alone would do the work in $\frac{75}{2} = 37\frac{1}{2}$ days.

Example 31. A and B together can do a piece of work in 12 days, which B and C together can do in 16 days. After A has been working at it for 5 days and B for 7 days, C finishes it in 13 days. In how many days C alone will do the work?

Solution.

A's 5 day's work + B's 7 day's work + C's 13 day's work =1

$\Rightarrow (A+B)'s\ 5\ day's\ work + (B+C)'s\ 2\ day's\ work + C's\ 11\ day's\ work = 1$

$\Rightarrow \frac{5}{12} + \frac{2}{16} + C's 11\ day's\ work = 1$

$\Rightarrow C's\ 11\ day's\ work = 1 - \left(\frac{5}{12} + \frac{2}{16}\right) = \frac{11}{24}$.

$\Rightarrow C's\ 1\ day's\ work = \left(\frac{11}{24} \times \frac{1}{11}\right) = \frac{1}{24}$.

$\therefore$ C alone can finish the work in 24 days.

Example 32. A, B and C can complete a work separately in 24, 36 and 48 days respectively. They started together but C left after 4 days of start and A left 3 days before the completion of the work. In how many days will the work be completed?

Solution.

$(A+B+C)'s$ 1 day's work $= \left(\frac{1}{24}+\frac{1}{36}+\frac{1}{48}\right) = \frac{13}{144}$.

Work done by $(A+B+C)$ in 4 days $= \left(\frac{13}{144}\times 4\right) = \frac{13}{36}$

Work done by B in 3 days $= \left(\frac{1}{36}\times 3\right) = \frac{1}{12}$.

Remaining work $= \left[1-\left(\frac{13}{36}+\frac{1}{12}\right)\right] = \frac{5}{9}$.

(A+B)'s 1 day's work $= \left(\frac{1}{24}+\frac{1}{36}\right) = \frac{5}{72}$.

Now, $\frac{5}{72}$ work is done by A and B in $\left(\frac{72}{5}\times\frac{5}{9}\right) = 8\ days$.

Hence, total time taken $= (4+3+8) days = 15\ days$.

Example 33. A sum of money is sufficient to pay A's wages for 21 days and B's wages for 28 days. The same money is sufficient to pay the wages of both for:

Solution.

Let total money be Rs. x.

A's 1 day's wages $= Rs.\frac{x}{21}$, B's 1 day's wages $= Rs.\frac{x}{28}$.

$\therefore (A+B)'s$ 1 day's wages $= Rs.\left(\frac{x}{21}+\frac{x}{28}\right) = Rs.\frac{x}{12}$.

$\therefore$ Money is sufficient to pay the wages of both for 12 days.

Example 34. A, B and C can do a piece of work in 11 days, 20 days and 55 days respectively, working alone. How soon can the work be done if A is assisted by B and C on alternate days?

Solution.

(A+B)'s 1 day's work $= \left(\frac{1}{11}+\frac{1}{20}\right) = \frac{31}{220}$.

(A+C)'s 1 day's work $= \left(\frac{1}{11}+\frac{1}{55}\right) = \frac{6}{55}$.

Work done in 2 days $= \left(\frac{31}{220}+\frac{6}{55}\right) = \frac{55}{220} = \frac{1}{4}$.

Now, $\frac{1}{4}$ work is done by A in 2 days.

$\therefore$ Whole work will be done in $(2\times 4) = 8\ days$.

Example 35. Seven men can complete a work in 12 days. They started the work and after 5 days, two men left. In how many days will the work be completed by the remaining men?

Solution.

(7×12) men can complete the work in 1 day.

$\therefore$1 man's 1 day's work $= \frac{1}{84}$.

7 men's 5 day's work $= \left(\frac{1}{12}\times 5\right) = \frac{5}{12}$.

Remaining work $= \left(1-\frac{5}{12}\right) = \frac{7}{12}$.

5 men's 1 day's work $= \left(\frac{1}{84}\times 5\right) = \frac{5}{84}$.

$\frac{5}{84}$ work is done by them in 1 day.

$\frac{7}{12}$ work is done by them in $\left(\frac{84}{5}\times\frac{7}{12}\right) = \frac{49}{5}\ days = 9\frac{4}{5}\ days$.

Example 36. Three men, four women and six children can complete a work in seven days. A woman does double the work a man does and a child does half the work a man does. How many women alone can complete this work in 7 days?

Solution.

Let 1 woman's 1 day's work = x.

Then, 1 man's 1 day's work $= \frac{x}{2}$ and 1 child's 1 day's work $= \frac{x}{4}$.

So, $\left(\frac{3x}{2} + 4x + \frac{6x}{4}\right) = \frac{1}{7} \quad \Rightarrow \frac{28x}{4} = \frac{1}{7} \quad \Rightarrow x = \left(\frac{1}{7} \times \frac{4}{28}\right) = \frac{1}{49}$.

∴ 1 woman alone can complete the work in 49 days.

So, to complete the work in 7 days, number of women required $= \left(\frac{49}{7}\right) = 7$.

Example 37. Twelve children take sixteen days to complete a work which can be completed by eight adults in twelve days. Sixteen adults started working and after three days ten adults left and four children joined them. How many days will they take to complete the remaining work?

Solution.

1 child's 1 day's work $= \frac{1}{192}$; 1 adult's 1 day's work $= \frac{1}{96}$.

Work done in 3 days $= \left(\frac{1}{96} \times 16 \times 3\right) = \frac{1}{2}$.

Remaining work $= \left(1 - \frac{1}{2}\right) = \frac{1}{2}$.

(6 adults + 4 children)'s 1 day's work $= \left(\frac{6}{96} + \frac{4}{192}\right) = \frac{1}{12}$.

$\frac{1}{12}$ work is done by them in 1 day.

$\frac{1}{2}$ work is done by them $\left(12 \times \frac{1}{2}\right) = 6\ days$.

Example 38. If 12 men and 16 boys can do a piece of work in 5 days; 13 men and 24 boys can do it in 4 days, then the ratio of the daily work done by a man to that of a boy is:

Solution.

Let 1 man's 1 day's work = x and 1 boy's 1 day's work = y.

Then, $12x + 16y = \frac{1}{5}\ and\ 13x + 24y = \frac{1}{4}$.

Solving these two equations, we get : $x = \frac{1}{100}\ and\ y = \frac{1}{200}$.

∴ Required ratio $= x : y = \frac{1}{100} : \frac{1}{200} = 2:1$.

EXERCISE

1. A can finish a work in 18 days and B can do the same work in half the time taken by A. Then, working together, what part of the same work they can finish in a day?
2. A tyre has two punctures. The first puncture alone would have made the tyre flat in 9 minutes and the second alone would have done it in 6 minutes. If air leaks out at a constant rate, how long does it take both the punctures together to make it flat?
3. A man can do a piece of work in 5 days, but with the help of his son, he can do it in 3 days. In what time can the son do it alone?
4. A can lay railway track between two given stations in 16 days and B can do the same job in 2 days. With the help of C, they did the job in 4 days only. Then, C alone can do the job in:
5. X can do $\frac{1}{4}$ of a work in 10 days, Y can do 40% of the work in 40 days and Z can do $\frac{1}{3}$ of the work in 13 days. Who will complete the work first?
6. Ronald and Elan are working on an assignment. Ronald takes 6 hours to type 32 pages on a computer, while Elan takes 5 hours to type 40 pages. How much time will they take, working together on two different computers to type an assignment of 110 pages?

7. A and B can do a work in 12 days, B and C in 15 days, C and A in 20days. If A, B and C work together, they will complete the work in :
8. A and B can do a work in 8 days, B and C can do the same work in 12 days. A, B and C together can finish it in 6 days. A and C together will do it in:
9. A and B can do a piece of work in 5 days; B and C can do it in 7 days; A and C can do it in 4 days. Who among these will take the least time if put to do it alone?
10. A can do a piece of work in 4 hours; B and C together can do it in 3 hours, while A and C together can do it in 2 hours. How long will B alone take to do it?
11. A works twice as fast as B. if B can complete a work in 12 days independently, the number of days in which A and B can together finish the work is:
12. A and B can do a job together in 7 days. A is $1\frac{3}{4}$ times as efficient as B. The same job can be done by A alone in:
13. Sakshi can do a piece of work in 20 days. Tanya is 25% more efficient than Sakshi. The number of days taken by Tanya to do the same piece of work is:
14. A does half as much work as B in three-fourth of the time. If together they take 18 days to complete the work, how much time shall B take to do it?
15. A can do a work in 15 days and B in 20 days. If they work on it together for 4 days, then the fraction of the work that is left is:
16. A can finish a work in 18 days and B can do the same work in 15 days. B worked for 10 days and left the job. In how many days, A alone can finish the remaining work?
17. A can finish a work in 24 days, B in 9 days and C in 12 days. B and C start the work but are forced to leave after 3 days. The remaining work was done by A in:
18. X and Y can do a piece of work in 20 days and 12 days respectively. X started the work alone and then after 4 days Y joined him till the completion of the work. How long did the work last?
19. X can do a piece of work in 40 days. He works at it for 8 days and then YP finished it in 16 days. How long will they together take to complete the work?
20. A, B and C together can complete a piece of work in 10 days. All the three started working at it together and after 4 days A left. Then B and C together completed the work in 10 more days. A alone could complete the work in:
21. A and B together can do a piece of work in 30 days. A having worked for 16 days, B finishes the remaining work alone in 44 days. In how many days shall B finish the whole work alone?
22. A and B can do a piece of work in 45 days and 40 days respectively. They began to do the work together but A leaves after some days and then B completed the remaining work in 23 days. The number of days after which A left the work was:
23. A can do a piece of work in 14 days which B can do in 21 days. They begin together but 3 days before the completion of the work, A leaves off. The total number of days to compete the work is:
24. A, B and C together earn Rs.300 per day, while A and C together earn Rs. 188 and B and C together earn Rs. 152. The daily earning of C is:
25. A, B and C are employed to do a piece of work for Rs. 529. A and B together are supposed to do $\frac{19}{23}$ of the work and B and C together $\frac{8}{23}$ of the work. What amount should A be paid?
26. Kim can do a work in 3 days while David can do the same work in 2 days. Both of them finish the work together and get Rs. 150. What is the share of Kim?
27. If A can do $\frac{1}{4}$ of a work in 3 days and B can do $\frac{1}{6}$ of the same work in 4 days, how much will A get if both work together and are paid Rs. 180 in all?

28. A alone can do a piece of work in 6 days and B alone in 8 days. A and B undertook to do it for Rs. 3200. With the help of C, they completed the work in 3 days. How much is to be paid to C?
29. A can do a piece of work in 10 days; B in 15 days. They work for 5 days. The rest of the work was finished by C in 2 days. If they get Rs. 1500 for the whole work, the daily wages of B and C are:
30. A and B together can complete a work in 12 days. A alone can complete it in 20 days. If B does the work only for half a day daily, then in how many days A and B together will complete the work?
31. A alone can complete a work in 16 days and B alone in 12 days. Starting with A, they work on alternate days. The total work will be completed in :
32. A,B and C can do a piece of work in 20,30 and 60 days respectively. In how many days can A do the work if he is assisted by B and C on every third day?
33. A and B can separately do a piece of work in 20 and 15 days respectively. They worked together for 6 days, after which B was replaced by C. If the work was finished in next 4 days, then the number of days in which C alone could do the work will be:
34. A, B and C can do a piece of work in 36,54 and 72 days respectively. They started the work but A left 8 days before the completion of the work while B left 12 days before the completion. The number of days for which C worked is:
35. Twenty women can do a work in sixteen days. Sixteen men can complete the same work in fifteen days. What is the ratio between the capacity of a man and a woman?
36. 10 men can complete a piece of work in 15 days and 15 women can complete the same work in 12 days. If all the 10 men and 15 women work together, in how many days will the work get completed?
37. 12 men complete a work in 9 days. After they have worked for 6 days, 6 more men join them. How many days will they take to complete the remaining work?
38. A man, a woman and a boy can complete a job in 3, 4 and 12 days respectively. How many boys must assist 1 man and 1 woman to complete the job in $\frac{1}{4}$ of a day?
39. 10 men and 15 women together can complete a work in 6 days. It takes 100 days for one man alone to complete the same work. How many days will be required for one woman alone to complete the same work?
40. 12 men can complete a piece of work in 4 days, while 15 women can complete the same work in 4 days. 6 men start working on the job and after working for 2 days, all of them stopped working. How many women should be put on the job to complete the remaining work, if it is to be completed in 3 days?
41. 10 women and can complete a work in 7 days and 10 children take 14 days to complete the work. How many days will 5 women and 10 children take to complete the work?
42. Sixteen men can complete a work in twelve days. Twenty-four children can complete the same work in eighteen days. Twelve men and eight children started working and after eight days three more children joined them. How many days will they now take to complete the remaining work?
43. Twenty-four men can complete a work in sixteen days. Thirty-two women can complete the same work in twenty-four days. Sixteen men and sixteen women started working and worked for twelve days. How many more men are to be added to complete the remaining work in 2 days?
44. 5 men and 2 boys working together can do four times as much work as a man and a boy. Working capacities of a woman and a boy are in the ratio:
45. 4 men and 6 women can complete a work in 8 days, while 3 men and 7 women can complete it in 10 days. In how many days will 10 women complete it?

46. One man, 3 women and 4 boys can do a piece of work in 96 hours, 2 men and 8 boys can do it in 80 hours, 2 men and 3 women can do it in 120 hours. 5 men and 12 boys can do it in:
47. If 6 men and 8 boys can do a piece of work in 10 days while 26 men and 48 boys can do the same in 2 days, the time taken by 15 men and 20 boys in doing the same type of work will be:

ANSWERS

1. $\frac{1}{6}$
2. $3\frac{3}{5}\ min$
3. $7\frac{1}{2}days$
4. $9\frac{3}{5}days$
5. Z
6. 8 hrs 15 min
7. 10 days
8. 8 days
9. A is least
10. 12 hrs
11. 4 days
12. $\frac{1}{11}$
13. 16 days
14. 30
15. $\frac{8}{15}$
16. 6 days
17. 10 days
18. 10 days
19. $13\frac{1}{3}$ days
20. 25 days
21. 60 days
22. A left after 9 days
23. $10\frac{1}{5}days$
24. Rs. 40
25. Rs. 345
26. Rs. 60
27. Rs. 120
28. Rs. 400
29. Rs. 225
30. 15 days
31. $13\frac{3}{4}$ days
32. 15 days
33. 40 days
34. 24
35. 4 : 3
36. $6\frac{2}{3}$ days
37. 2 days
38. 41
39. 225 days
40. 15 women
41. 7 days
42. 4 days
43. 24 men
44. 2 : 1
45. 40 days
46. $43\frac{7}{11}$ hours
47. 4 days

UNIT III
TIME AND DISTANCE

SOME IMPORTANT FACT AND FORMULAE

1. Speed = $\left(\frac{Distance}{Time}\right), Time = \left(\frac{Distance}{Speed}\right), Distance = (Speed \times Time)$
2. $x\ km/hr\ = \left(x \times \frac{5}{18}\right)\ m/sec$
3. $x\frac{m}{sec} = \left(x \times \frac{18}{5}\right) km/hr$
4. If the ratio of the speeds of A and B is a : b, then the ratio of the times taken by them to cover the same distance is $\frac{1}{a} : \frac{1}{b}\ or\ b : a$.
5. Suppose a man covers a certain distance at $x\ km/hr$ and an equal distance at $y\ km/hr$. Then, the average speed during the whole journey is $\left(\frac{2xy}{x+y}\right)\ km/hr$.

SOLVED PROBLEMS

Example 1. How many minutes does Aditya take to cover a distance of 400m, if he runs at a speed of 20 km/hr?

Solution.

Aditya's speed = 20 km/hr $= \left(20 \times \frac{5}{18}\right) m/sec\ = \frac{50}{9}\ m/sec.$

$\therefore$ Time taken to cover 400 m $= \left(400 \times \frac{9}{50}\right) sec = 72\ sec$

$= 1\frac{12}{60}\ min$

$= 1\frac{1}{5} min.$

Example 2. A cyclist covers a distance of 750 m in 2 min 30 sec. What is the speed in km/hr of the cyclist?

Solution.

Speed $= \left(\frac{750}{150}\right)\ m/sec$

$= 5 m/sec$

$= \left(5 \times \frac{18}{5}\right)\ km/hr$

$= 18\ km/hr.$

Example 3. A dog takes 4 leaps for every 5 leaps of a hare but 3 leaps of a dog are equal to 4 leaps of the hare. Compare their speeds.

Solution.

Let the distance covered in 1 leap of the dog be x and that covered in 1 leap of the here be y.

Then, $3x = 4y \quad \Rightarrow x = \frac{4}{3}y$

$\Longrightarrow 4x = \frac{16}{3}y.$

∴ Ratio of speeds of dog and hare = Ratio of distances covered by them in the same time

$$= 4x : 5y = \frac{16}{3}y : 5y = \frac{16}{3} : 5$$

$$= 16 : 15$$

Example 4. While covering a distance of 24 km, a man noticed that after walking for 1 hour and 40 minutes, the distance covered by him was $\frac{5}{7}$ of the remaining distance. What was his speed in meters per second?

Solution.

Let the speed be x km/hr.

Then, distance covered in 1 hr. 40 min. i.e., $1\frac{2}{3} hrs = \frac{5x}{3} km.$

Remaining distance $= \left(24 - \frac{5x}{3}\right) km$

$\therefore \frac{5x}{3} = \frac{5}{7}\left(24 - \frac{5x}{3}\right)$

$\Leftrightarrow \frac{5x}{3} = \frac{5}{7}\left(\frac{72-5x}{3}\right)$

$\Leftrightarrow 7x = 72 - 5x$

$\Leftrightarrow 12x = 72$

$\Leftrightarrow x = 6$

Hence, speed $= \frac{6km}{hr} = \left(6 \times \frac{5}{18}\right) m/sec$

$$= \frac{5}{3} m/sec$$

$$= 1\frac{2}{3} m/sec.$$

Example 5. Peter can cover a certain distance in 1 hr. 24 min. by covering two-third of the distance at 4 kmph and the rest at 5 kmph. Find the total distance.12

Solution.

Let the total distance be x km. Then,

$$\frac{\frac{2}{3}x}{4} + \frac{\frac{1}{3}x}{5} = \frac{7}{5}$$

$$\Leftrightarrow \frac{x}{16} + \frac{x}{15} = \frac{7}{5}$$

$$\Leftrightarrow 7x = 42$$

$$\Leftrightarrow x = 6$$

Example 6. A man travelled from the village to the post-office at the rate of 25 kmph and walked back at the rate of 4 kmph. If the whole journey took 5 hours 48 minutes. Find the distance of the post-office from the village.

Solution.

Average speed $= \left(\frac{2xy}{x+y}\right)\ km/hr$

$= \left(\frac{2\times25\times4}{25+4}\right)\ km/hr$

$= \frac{200}{29}\ km/hr$

Distance travelled in 5 hours 48 minutes

i.e, $5\frac{4}{5}\ hrs = \left(\frac{200}{29}\times\frac{29}{5}\right)km\ = 40\ km.$

∴ Distance of the post-office from the village $= \left(\frac{40}{2}\right) = 20km.$

Example 7. An aero plane flies along the four sides of a square at the speeds of 200, 400, 600 and 800 km/hr. Find the average speed of the plane around the field.

Solution.

Let each side of the square be x km and let the average speed of the plane around the field be y km/hr. Then,

$\frac{x}{100}+\frac{x}{400}+\frac{x}{600}+\frac{x}{800}=\frac{4z}{y}$

$\Leftrightarrow \frac{25x}{2400}=\frac{4x}{y}$

$\Leftrightarrow y = \left(\frac{2400\times4}{25}\right) = 384.$

∴ Average speed = 384 km/hr.

Example 8. Walking at $\frac{5}{6}$ of its usual speed, a train is 10 minutes too late. Find its usual time to cover the journey.

Solution.

New speed $= \frac{5}{6}$ of the usual speed

∴ New time taken $= \frac{6}{5}$ of the usual time

So, $\left(\frac{6}{5}\ of\ the\ usual\ time\right) - (usual\ time) = 10\ min.$

$\Rightarrow \frac{1}{5}\ of\ the\ usual\ time = 10\ min$

$\Longrightarrow usual\ time = 50\ min.$

Example 9. If a man walks at the rate of 5 kmph, he misses a train by 7 minutes. However, if he walks at the rate of 6 kmph, he reaches the station 5 minutes before the arrival of the train. Find the distance covered by him to reach the station.

Solution.

Let the required distance be x km.

Difference in the times taken at two speeds $= 12\ min = \frac{1}{5}\ hr.$

$\therefore \frac{x}{5} - \frac{x}{6} = \frac{1}{5} \quad \Leftrightarrow 6x - 5x = 6$

$\Leftrightarrow x = 6.$

Hence, the required distance is 6 km.

Example 10. A and B are two stations 390 km apart. A train starts from A at 10 a.m. and travels towards B at 65 kmph. Another train starts from B at 11 a.m. and travels towards A at 35kmph. At what time do they meet?

Solution.

Suppose they meet x hours after 10 a.m. Then,

(Distance moved by first in x hrs) + [Distance moved by second in (x-1) hrs] =390

$\therefore 65x + 35(x - 1) = 390 \ \Rightarrow 100x = 425$

$\Longrightarrow x = 4\frac{1}{4}.$

So, they meet 4 hrs, 15 min, after 10 a.m. i.e., at 2.15 p.m.

Example 11. A goods train leaves a station at a certain time and at a fixed speed. After 6 hours, an express train leaves the same station and moves in the same direction at a uniform speed of 90 kmph. This train catches up the goods train in 4 hours. Find the speed of the goods train.

Solution.

Let the speed of the goods train be x kmph.

Distance covered by goods train in 10 hours = Distance covered by express train in 4 hours

$\therefore 10x = 4 \times 90\ or\ x = 36.$

So, speed of goods train = 36 kmph.

Example 12. A thief is spotted by a policeman from a distance of 100 meters. When the policeman starts the chase, the thief also starts running. If the speed of the thief be 8 km/hr and that of the policeman 10 km/hr, how far the thief will have run before he his overtaken?

Solution.

Relative speed of the policeman $= (10 - 8) km/hr \ = 2 \ km/hr.$

Time taken by policeman to cover $100 \ m = \left(\frac{100}{1000} \times \frac{1}{2}\right) hr = \frac{1}{20} hr.$

In $\frac{1}{20} hrs$, the thief covers a distance of $\left(8 \times \frac{1}{20}\right) km = \frac{2}{5} km = 400 \ m.$

Example 13. I walk a certain distance and rid back taking a total time of 37 minutes. I could walk both ways in 55 minutes. How long would it take me to ride both ways?

Solution.

Let the distance be x km. Then,

(Time taken to walk x km) + (Time taken to ride x km) = 37 min.

⇒ (Time taken to walk 2x km) + (Time taken to ride 2x km) = 74 min.

But, time taken to walk 2x km = 55 min.

∴ Time taken to ride 2x km $= (74 - 55)$ min $= 19 \ min.$

Example 14. A car moves at the speed of 80 km/hr. What is the speed of the car in meters per second?

Solution.

Speed $= \left(80 \times \frac{5}{18}\right) \ m/sec$

$= \frac{200}{9} m/sec$

$= 22\frac{2}{9} m/sec.$

Example 15. How long will a boy take to run round a square field of side 35 meters, if he runs at the rate of 9 km/hr?

Solution.

Speed $= 9 \ km/hr$

$= \left(9 \times \frac{5}{18}\right)$ m/sec

$= \frac{5}{2}$ m/sec.

Distance $= (35 \times 4) m = 140 \ m.$

$\therefore \ Time \ taken = \left(140 \times \frac{2}{5}\right) sec$

$= 56 \ sec.$

Example 16. One of the two buses completes a journey of 300 km in $7\frac{1}{2}$ hours and the other a journey of 450km in 9 hours. The ratio of their average speeds is:

Solution.

$$\text{Ratio of speeds} = \left(300 \times \frac{2}{15}\right) : \left(\frac{450}{9}\right)$$

$$= 40 : 50$$

$$= 4 : 5$$

Example 17. Sound is said to travel in air at about 1100 feet per second. A man hears the axe striking the tree, $\frac{11}{5}$ seconds after he sees it strike the tree. How far is the man from the wood chopper?

Solution.

$$\text{Distance} = \left(1100 \times \frac{11}{5}\right) \; feet.$$

$$= 2420 \text{ feet.}$$

Example 18. An express train travelled at an average speed of 100 km/hr, stopping for 3 minutes after every 75 km. How long did it take to reach its destination 600 km from the starting point?

Solution.

$$\text{Time taken to cover } 600 \; km = \left(\frac{600}{100}\right) hrs = 6 \; hrs.$$

$$\text{Number of stoppages} = \frac{600}{75} - 1 = 7.$$

$$\text{Total time of stoppage} = (3 \times 7) \text{ min} = 21 \; min.$$

$$\text{Hence, total time taken} = 6 \; hrs \; 21 \; min$$

Example 19. A certain distance is covered by a cyclist at a certain speed. If a jogger covers half the distance in double the time, the ration of the speed of the jogger to that of the cyclist is :

Solution.

Let the distance covered by the cyclist be x and the time taken be y. Then,

$$\text{Required ratio} = \frac{\frac{1}{2}x}{2y} : \frac{x}{y}$$

$= \frac{1}{4}$

$= 1 : 4$

Example 20. The speed of a car increases by 2 kms after one hour. If the distance travelled in the first one hour was 35 kms, what was the total distance travelled in 12 hours?

Solution.

Total distance travelled in 12 hours $= (35 + 37 + 39 + \cdots \ldots \ldots upto\ 12\ terms)$

This is an A.P. with first term, a= 35, number of terms, n = 12,

Common difference, d=2.

$\therefore$ Required distance $= \frac{12}{2}[2 \times 35 + (12 - 1) \times 2]$

$= 6(70 + 22) = 552\ km.$

Example 21. A salesman travels a distance of 50 km in 2 hours and 30 minutes. How much faster, in kilometers per hour, on an average, must be travel to make such a trip in $\frac{5}{6}$ hour less time?

Solution.

Time required $= (2\ hrs\ 30\ \text{min} - 50\ \text{min}) = 1\ hr\ 40\ \text{min}$

$= 1\frac{2}{3}$ hrs.

$\therefore$ Required Speed $= \left(50 \times \frac{3}{5}\right)\ km/hr$

$= 30\ km/hr.$

Original speed $= \left(50 \times \frac{2}{5}\right) km/hr$

$= 20\ km/hr$

$\therefore$ Difference in speed $= (30 - 20) km/hr$

$= 10\ km/hr.$

Example 22. A can complete a journey in 10 hours. He travels first half of the journey at the rate of 21 km/hr and second half at the rate of 24 km/hr. Find the total journey in km.

Solution.

Let the total distance be x km. Then,

$$\frac{\frac{1}{2}x}{21} + \frac{\frac{1}{2}x}{24} = 10$$

$$\Rightarrow \frac{x}{21} + \frac{x}{24} = 20$$

$$\Longrightarrow 15x = 168 \times 20$$

$$\Longrightarrow x = \left(\frac{168\times20}{15}\right) = 224\ km.$$

Example 23. A is faster than B. A and B each walk 24 km. The sum of their speeds is 7 km/hr and the sum of times taken by them is 14 hours. Find A's speed.

Solution.

Let A's speed = x km/hr.

Then, B's speed $= (7 - x) km/hr$.

So, $\frac{24}{x} + \frac{24}{(7-x)} = 14$

$$\Leftrightarrow 24\,(7 - x) + 24x = 14x(7 - x)$$

$$\Leftrightarrow 14\,x^2 - 98x + 168 = 0$$

$$\Leftrightarrow x^2 - 7x + 12 = 0$$

$$\Leftrightarrow (x - 3)(x - 4) = 0$$

$$\Leftrightarrow x = 3\ or\ x = 4.$$

Since, A is faster than B. so A's speed $= 4\ km/hr$ and B's speed $= 3\ km/hr$.

Example 24. Mac travels from A to B a distance of 250 miles in $5\frac{1}{2}$ hours. He returns to A in 4 hours 30 minutes. Find His average speed.

Solution.

Speed from A to B $= \left(250 \times \frac{2}{11}\right)\ mph = \left(\frac{500}{11}\right) mph.$

Speed from B to A $= \left(250 \times \frac{2}{9}\right) mph = \left(\frac{500}{9}\right) mph.$

$$\therefore \text{Average speed} = \left(\frac{2\times\frac{500}{11}\times\frac{500}{9}}{\frac{500}{11}+\frac{500}{9}}\right) mph$$

$$= \left(\frac{500000}{4500+5500}\right) mph$$

$$= 50\ mph$$

Example 25. I started on my bicycle at 7 a.m to reach a certain place. After going a certain distance, my bicycle went out of order. Consequently, I rested for 35 minutes and came back

to my house walking all the way. I reached my house at 1 p.m. If my cycling speed is 10 kmph and my walking speed is 1 kmph, then on my bicycle I covered a distance of:

Solution.

Time taken $= 5hrs\ 25\ \text{min} = \frac{65}{12}\ hrs.$

Let the required distance be x km.

Then,

$\frac{x}{10} + \frac{x}{1} = \frac{65}{12}$

$\Leftrightarrow 11x = \frac{650}{12}$

$\Leftrightarrow x = \frac{325}{66}$

$= 4\frac{61}{66} km.$

Example 26. A, B and C are on a trip by a car. A drives during the first hour at an average speed of 50 km/hr. B drives during the next 2 hours at an average speed of 48 km/hr. C drives for the next 3 hours at an average speed of 52 km/hr. They reached their destination after exactly 6 hours. Their mean speed was:

Solution.

Total distance travelled $= (50 \times 1 + 48 \times 2 + 52 \times 3)km = 302\ km.$

Total time taken $= 6\ hrs.$

$\therefore$ Mean speed $= \left(\frac{302}{6}\right) km/hr$

$= 50\frac{1}{3} km/hr$

Example 27. A car travels the first one-third of a certain distance with a speed of 10 km/hr, the next one-third distance with a speed of 20 km/hr, and the last one-third distance with a speed of 60 km/hr. The average speed of the car for the whole journey is:

Solution.

Let the whole distance travelled be x km and the average speed of the car for the whole journey be y km/hr.

Then,

$\left(\frac{(x/3)}{10}\right) + \frac{(x/3)}{20} + \frac{(x/3)}{60} = \frac{x}{y}$

$\Leftrightarrow \frac{x}{30} + \frac{x}{60} + \frac{x}{180} = \frac{x}{y}$

$\Leftrightarrow \frac{1}{18}y = 1 \Leftrightarrow y = 18\ km/hr.$

Example 28. Mary jogs 9 km at a speed of 6 km per hour. At what speed would she need to jog during the next 1.5 hours to have an average of 9 km per hour for the entire jogging session?

Solution.

Let speed of jogging be x km/hr.

Total time taken $= \left(\frac{9}{6}hrs + 1.5\ hrs\right) = 3hrs.$

Total distance covered $= (9 + 1.5x)km.$

$\therefore \frac{9+1.5x}{3} = 9$

$\Leftrightarrow 9 + 105x = 27$

$\Leftrightarrow \frac{3}{2}x = 18$

$\Leftrightarrow x = \left(18 \times \frac{2}{3}\right) = 12\ kmph.$

Example 29. Walking $\frac{6}{7}th$ of his usual speed, a man is 12 minutes too late. Then, what is the usual time taken by him to cover that distance?

Solution.

New speed $= \frac{6}{7}$ of usual speed.

New time $= \frac{7}{6}$ of usual time

$\therefore \left(\frac{7}{6}of\ usual\ time\right) - (usual\ time) = \frac{1}{5}hr.$

$\Rightarrow \frac{1}{6}\ of\ usual\ time = \frac{1}{5}hr$

$\Longrightarrow usual\ time = \frac{6}{5}hr = 1hr\ 12\ min.$

Example 30. Robert is travelling on his cycle and has calculated to reach point A at 2 P.M. if he travels at 10 kmph; he will reach there at 12 noon if he travels at 15 kmph. At what speed must he travel to reach A at 1 P.M.?

Solution.

Let the distance travelled be x km.

Then, $\frac{x}{10} - \frac{x}{15} = 2$

$\Leftrightarrow 3x - 2x = 60$

$\Leftrightarrow x = 60\ km.$

Time taken to travel 60 km at 10 km/hr $= \left(\frac{60}{10}\right) hrs = 6\ hrs.$

So, Robert started 6 hours before 2 P.M. i.e., at 8 A.M.

∴ Required speed $= \left(\frac{60}{5}\right) kmph = 12\ kmph.$

Example 31. A car travel from P to Q at a constant speed. If its speed were increased by 10 km/hr. it would have taken one hour lesser to cover the distance. It would have taken further 45 minutes lesser if the speed was further increased by 10 km/hr. What is the distance between the two cities?

Solution.

Let distance = x km and usual rate = y kmph. Then,

$\frac{x}{y} - \frac{x}{y+10} = 1$ or $y(y+10) = 10x$ ... ... (i)

And, $\frac{x}{y} - \frac{x}{y+20} = \frac{7}{4}$ $or\ y(y+20) = \frac{80x}{7}$ ... ... (ii)

On dividing (i) by (ii), we get y = 60.

Substituting y = 60 in (i), we get : x = 420 km.

Example 32. A train can travel 50% faster than a car. Both start from point A at the same time and reach point B 75 kms away from A at the same time. On the way, however, the train lost about 12.5 minutes while stopping at the stations. The speed of the car is:

Solution.

Let speed of the car be x kmph.

Then, speed of the train $= \frac{500}{100}x = \left(\frac{3}{2}x\right) kmph.$

∴ $\frac{75}{x} - \frac{75}{\frac{3}{2}x} = \frac{125}{10\times60}$

$\Leftrightarrow \frac{75}{x} - \frac{50}{x} = \frac{5}{24}$

$\Leftrightarrow x = \left(\frac{25\times24}{5}\right) = 120\ kmph.$

Example 33. A car covers a distance of 715 km at a constant speed. If the speed of the car would have been 10 km/hr more, then it would have taken 2 hours less to cover the same distance. What is the original speed of the car?

Solution.

Let the original speed be x km/hr. Then,

$\frac{715}{x} - \frac{715}{x+10} = 2$

$\Leftrightarrow 2x(x+10) = 7150$

$\Leftrightarrow x^2 + 10x - 3575 = 0$

$\Leftrightarrow (x+65)(x-55) = 0$

$\Leftrightarrow x = 55\ km/hr$

Example 34. In covering a certain distance, the speeds of A and B are in the ration of 3 : 4. A takes 30 minutes more than B to reach the destination. The time taken by A to reach the destination is:

Solution.

Ratio of speeds = 3 : 4.

Ratio of times taken = 4 : 3

Suppose A takes 4x hrs and B takes 3x hrs to reach the destination. Then,

$4x - 3x = \frac{30}{60} = \frac{1}{2}$ or $x = \frac{1}{2}$.

$\therefore$ Time taken by $A = 4x\ hrs\ = \left(4 \times \frac{1}{2}\right) hrs = 2\ hrs.$

Example 35. Two men start together to walk to a certain destination, one at 3 kmph and another at 3.75 kmph. The latter arrives half an hour before the former. The distance is:

Solution.

Let the distance be x km. Then,

$\frac{x}{3} - \frac{x}{3.75} = \frac{1}{2}$

$\Leftrightarrow 2.5x = 2x = 3.75$

$\Leftrightarrow x = \frac{3.75}{0.50} = \frac{15}{2} = 7.5\ km.$

Example 36. A is twice as fast as B and B is thrice as fast as C is. The journey covered by C in 54 minutes will be covered by B in:

Solution.

Let C's speed = x km/hr.

Then, B's speed = 3x km/hr and A's speed = 6x km/hr.

∴ Ratio of speeds of A, B, C = 6x : 3x : x = 6 : 3 : 1

Ratio of times taken $= \frac{1}{6} : \frac{1}{3} : 1 = 1 : 2 : 6.$

If C takes 6 min, then B takes 2 min.

If C takes 54 min, then B takes $\left(\frac{2}{6} \times 54\right)$ min $= 18\ min$

Example 37. A thief steals a car tat 2.30 p.m and drives it at 60 kmph. The theft is discovered at 3 p.m. and the owner sets off in another car at 75 kmph. When will he overtake the thief?

Solution.

Suppose the thief is overtaken x hrs after 2.30 p.m

Then, distance covered by the thief in x hrs = distance covered by the owner in $\left(x - \frac{1}{2}\right)$hrs.

$\therefore 60x = 75\ \left(x - \frac{1}{2}\right)$

$\Leftrightarrow 15x = \frac{75}{2}$

$\Leftrightarrow x = \frac{5}{2}$ hrs.

So, the thief is overtaken at 5 p.m.

Example 38. Two cyclists start form the same place in opposite directions. One goes towards north at 18 kmph and the other goes towards south at 20 kmph. What time will they take to be 47.5 km apart?

Solution.

To be $(18 + 20)km$ apart, they take 1 hour.

To be 47.5 km apart, they take $\left(\frac{1}{38} \times 47.5\right) hrs = 1\frac{1}{4} hrs.$

Example 39. Two cars P and Q start at the same time from A and B which are 120 km apart. If the two cars travel in opposite directions, they meet after one hour and if they travel in same direction (from A towards B), then P meets Q after 6 hours. What is the speed of car P?

Solution.

Let their speeds be x kmph and y kmph respectively.

Then, $\frac{120}{x+y} = 1 \quad \Rightarrow x + y = 120 \qquad \ldots\ldots (i)$

Now, when they move in same direction:

(Distance travelled by P in 6 hrs) – (Distance travelled by Q in 6 hrs) = 120 km

$\Longrightarrow 6x - 6y = 120 \quad \Longrightarrow x - y = 20 \qquad \ldots\ldots (ii)$

Solving (i) and (ii), x=70, y = 50.

$\therefore$ P's speed $= 70\ kmph.$

Example 40. Two trains start from P and Q respectively and travel towards each other at a speed of 50 km/hr and 40 km/hr respectively. By the time they meet, the first train has travelled 100 km more than the second. The distance between P and Q is:

Solution.

At the time of meeting, let the distance travelled by the second train be x km.

Then, distance covered by the first train is $(x + 100)$ km.

$\therefore \frac{x}{40} = \frac{x+100}{50}$

$\Leftrightarrow 50x = 40x + 4000$

$\Leftrightarrow x = 400$

So, distance between P and Q $= (x + x + 100)km = 900\ km.$

Example 41. A man takes 5 hours 45 min in walking to a certain place and riding back. He would have gained 2 hours by riding both ways. The time he would take to walk both ways, is:

Solution.

Let the distance be x km. Then,

(Time taken to walk x km) +(Time taken to ride x km) $= \frac{23}{4} hrs.$

$\Longrightarrow$ (Time taken to walk 2x km) + (Time taken to ride 2x km) $= \frac{23}{2}\ hrs.$

But, time taken to ride 2x km $= \frac{15}{4} hrs.$

$\therefore$ Time taken to walk 2x km $= \left(\frac{23}{2} - \frac{15}{4}\right)$hrs

$= \frac{31}{4} hrs$

$= 7\ hrs\ 45\ min.$

EXERCISE

1. An athlete runs 200 meters race in 24 seconds. What is his speed ?
2. A person crosses a 600 m long street in 5 minutes. What is his speed in km per hour?
3. A man walking at the rate of 5 km / hr crosses a bridge in 15 minutes. The length of the bridge (in meters) is:
4. A car is running at a speed of 108 kmph. What distance will it cover in 15 seconds?
5. A truck covers a distance of 550 meters in 1 minute whereas a bus covers a distance of 33 kms in 45 minutes. The ratio of their speeds is:
6. The ratio between the speeds of two trains is 7 : 8. If the second train runs 400 kms in 4 hours, then the speed of the first train is:
7. A train travels at an average of 50 miles per hour for $2\frac{1}{2}$ hours and then travels at a speed of 70 miles per hour for $1\frac{1}{2}$ hours. How far did the train travel in the entire 4 hours?
8. A man in a train notices that he can count 21 telephone posts in one minute. If they are known to be 50 meters apart, then at what speed is the train travelling?
9. A motor car starts with the speed of 70 km/hr with its speed increasing every two hours by 10 kmph. In how many hours will it cover 345 kms?
10. A train covers a distance of 10 km in 12 minutes. If its speed is decreased by 5 km/hr, the time taken by it to cover the same distance will be:
11. Anna left for city A from city B at 5.20 a.m. She travelled at the speed of 80 km/hr for 2 hours 15 minutes. After that the speed was reduced to 60 km/hr. If the distance between two cities is 350 kms, at what time did Anna reach city A?
12. An aeroplane covers a certain distance at a speed of 240 kmph in 5 hours. To cover the same distance in $1\frac{2}{3}$ hours, it must travel at a speed of:
13. A person has to cover a distance of 6 km in 45 minutes. If he covers one-half of the distance in two-thirds of the total time; to cover the remaining distance in the remaining time, his speed (in km/hr) must be:
14. A man performs $\frac{3}{5}$ of the total journey by rail, $\frac{17}{20}$ by bus and the remaining 6.5km on foot. His total journey is:
15. A person travels equal distances with speeds of 3km / hr, 4km/hr and 5km / hr and takes a total time of 47 minutes. The total distance (in km) is :
16. A farmer travelled a distance of 61km in 9 hours. He travelled partly on foot @ 4km / hr and partly on bicycle @9km /hr. The distance travelled on foot is:
17. A person travels from P to Q at a speed of 40kmph and returns by increasing his speed by 50%. What is his average speed for both the trips?
18. A car driver travels from the plains to the hill station, which are 200km apart at an average speed of 40km / hr. In the return trip, he covers the same distance at an average speed of 20km/hr. The average speed of the car over the entire distance of 400km is:
19. A boy goes to his school from his house at a speed of 3km/hr and returns at a speed of 2km /hr. If he takes 5hours in going and coming, the distance between his house and school is:
20. The average speed of a train in the onward journey is 25% more than that in the return journey. The train halts for one hour on reaching the destination. The total time taken

for the complete to and fro journey is 17 hours, covering a distance of 800km. The speed of the train in the train in the onward journey is:

21. A man on tour travels first 160km at 64km /hr and the next 160km at 80km / hr. The average speed for the first 320km of the tour is:
22. A boy rides his bicycle 10 km at an average speed of 12 km/hr and again travels 12 km at an average speed of 10 km/hr. His average speed for the entire trip is approximately.
23. A man travels 600 km by train at 80 km/hr, 800 km by ship at 40 km/hr, 500 km by aeroplane at 400 km/hr and 100 km by car at 50 km/hr. What is the average speed for the entire distance?
24. A motorist covers a distance of 39 km in 45 minutes by moving at a speed of x kmph for the first 15 minutes, then moving at double the speed for the next 20 minutes and then again moving at his original speed for the rest of the journey. Then, x is equal to:
25. A car travelling with $\frac{5}{7}$ of its actual speed covers 42 km in 1 hr 40 min 48 sec. Find the actual speed of the car.
26. A train running at $\frac{7}{11}$ of its own speed reached a place in 22 hours. How much time could be saved if the train would have run at its own speed?
27. A man can reach a certain place in 30 hours. If he reduces his speed by $\frac{1}{15}th,$ he goes 10km less in that time. Find his speed.
28. Starting from his house one day, a student walks at a speed of $2\frac{1}{2}kmph$ and reaches his school 6 minutes late. Next day he increases his speed by 1 kmph and reaches the school 6 minutes early. How far is the school from his house?
29. A train when moves at an average speed of 40 kmph, reaches its destination on time. When its average speed becomes 35 kmph, then it reaches its destination 15 minutes late. Find the length of journey.
30. If a train runs at 40 kmph, it reaches its destination late by 11 minutes but if it runs at 50 kmph, it is late by 5 minutes only. The correct time for the train to complete its journey is:
31. A man covered a certain distance at some speed. Had he moved 3 kmph faster, he would have taken 40 minutes less. If he had moved 2 kmph slower, he would have taken 40 minutes more. the distance (in km) is;
32. Excluding stoppages, the speed of a bus is 54 kmph and including stoppages, it is 45 kmph. For how many minutes does the bus stop per hour?
33. In covering a distance of 30 km, Abhay takes 2 hours more than Sameer. If Abhay doubles his speed, then he would take 1 hour less than Sameer. Abhay's speed is:
34. Three persons are walking from a place A to another place B. Their speeds are in the ratio of 4 : 3: 5. The time ratio to reach B by these persons will be:
35. With a uniform speed a car covers the distance in 8 hours. Had the speed been increased by 4 km/hr, the same distance could have been covered in $7\frac{1}{2}$ hours. What is the distance covered?
36. If a person walks at 14 km/hr instead of 10 km/hr, he would have walked 20 km more. The actual distance travelled by him is:
37. In a flight of 600 km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km/hr and the time of flight increased by 30 minutes. The duration of the flight is:

38. It takes eight hours for a 600 km journey, if 120 km is done by train and the rest by car. It takes 20 minutes more, if 200 km is done by train and the rest by car. The ratio of the speed of the train to that of the car is:
39. Two men starting from the same place walk at the rate of 5 kmph and 5.5 kmph respectively. What time will they take to be 8.5 km apart, if they walk in the same direction?
40. A walks around a circular field at the rate of one round per hour while B runs around it at the rate of six rounds per hour. They start in the same direction from the same point at 7.30 a.m. They shall first cross each other at:
41. A walks at 4 kmph and 4 hours after his start, B cycles after him at 10 kmph. How far from the start does B catch up with A?
42. A thief is noticed by a policeman from a distance of 200 m. The thief starts running and the policeman chases him. The thief and the policeman run at the rate of 10km and 11 km per hour respectively. What is the distance between them after 6 minutes?
43. Two guns were fired from the same place at an interval of 10 minutes and 30 seconds, but a person in the train approaching the place hears the second shot 10 minutes after the first. The speed of the train (in km/hr), supposing that speed travels at 330 meters per second, is:
44. The distance between two cities A and B is 330 km. A train starts from A at 8 a.m. and travels towards B at 60 km/hr. Another train starts from B at 9 a.m. and travels towards A at 75 km/hr. At what time do they meet?
45. The jogging track in a sports complex is 726 meters in circumference. Deepak and his wife start from the same point and walk in opposite directions at 4.5 km/hr and 3.75 km/hr respectively. They will meet for the first time in:
46. A and B walk around a circular track. They start at 8 a.m. from the same point in the opposite directions. A and B walk at a speed of 2 rounds per hour and 3 rounds per hour respectively. How many times shall they cross each other before 9.30 a.m.?
47. Two trains starting at the same time from two stations 200 km apart and going in opposite directions cross each other at a distance of 110 km from one of the stations. What is the ratio of their speeds?
48. Bombay Express left Delhi for Bombay at 14.30 hrs, travelling at a speed of 60 kmph and Rajdhani Express left Delhi for Bombay on the same day at 16.30 hrs, travelling at a speed of 80 kmph. How far away from Delhi will the two trains meet?
49. A train M leaves Meerut at 5 a.m. and reaches Delhi at 9 a.m. Another train leaves Delhi at 7 a.m. and reaches Meerut at 10.30 a.m. At what time do the two trains cross each other?

ANSWERS

1. 30 km/hr
2. 7.2 km/hr
3. 1250 m
4. 450 m
5. 3 : 4
6. 87.5 km/hr
7. 230 miles
8. 60 km/hr
9. $4\frac{1}{2}$ hrs
10. 13 min 20 sec
11. 10.25 a.m
12. 720 km/hr
13. 12 km/hr
14. 130 km
15. 3 km
16. 16 km
17. 48 km/hr
18. 26.67km/hr
19. 6 km
20. 56.25 km/hr
21. 71.11 km/hr
22. 10.8 km/hr
23. $65\frac{5}{123}$ hr
24. x= 36
25. 35 km/hr
26. 8 hrs
27. 5 km/hr
28. $1\frac{3}{4}$km
29. 70 km
30. 19 min
31. 40 km
32. 10 min
33. 5 km/hr
34. 15 : 20 : 12
35. 480 km
36. 50 km
37. 1 hr
38. 3 : 4
39. 17 hrs
40. 12 min
41. 26.7 km
42. 100 m
43. 59.4 km/hr
44. 11 a.m
45. 5.28 min
46. 9.30 a.m
47. 11 : 9
48. 480 km
49. 7.56 a.m

UNIT IV
CHAIN RULE

1. **Direct Proportion :** Two quantities are said to be directly proportional, if on the increase (or decrease) of the one, the other increases (or decreases) to the same extent.
   Ex 1. Cost is directly proportional to the number of articles.
   (More Articles, More Cost)
   Ex 2. Work done is directly proportional to the number of men working on it.
   (More Men, More Work)
2. **Indirect Proportion :** Two quantities are said to be indirectly proportional, if on the increase of the one, the other decreases to the same extent and vice-versa.
   Ex 1. The time taken by a car in covering a certain distance is inversely proportional to the speed of the car.
   (More speed, Less is the time taken to cover a distance)
   Ex 2. Time taken to finish a work is inversely proportional to the number of persons working at it.
   (More persons, Less is the time taken to finish a job)
   Remark: In solving questions by chain rule, we compare every item with the term to be found out.

SOLVED EXAMPLES

Example 1. If 15 toys cost Rs. 234, what do 35 toys cost?

Solution.

Let the required cost be Rs. X. Then,

More toys, More cost **(Direct Proportion)**

$\therefore 15 : 35 :: 234 : x$

$\Leftrightarrow (15 \times x) = (35 \times 234)$

$\Leftrightarrow x = \left(\frac{35 \times 234}{15}\right) = 546.$

Hence, the cost of 35 toys is Rs. 546.

Example 2. If 36 men can do a piece of work in 25 hours, in how many hours will 15 men do it?

Solution.

Let the required number of hours be x. Then,

Less men, More hours **(Indirect proportion)**

$\therefore 15 : 36 :: 25 : x$

$\Leftrightarrow (15 \times x) = (36 \times 25)$

$\Leftrightarrow x = \frac{36 \times 25}{15} = 60.$

Hence, 15 men can do it in 60 hours.

Example 3. If the wages of 6 men for 15 days be Rs. 2100, then find the wages of 9 men for 12 days.

Solution.

Let the required wages be Rs. x.

More men, More wages **(Direct Proportion)**

Less days, Less wages **(Direct Proportion)**

$\left.\begin{matrix} Men\ 6 : 9 \\ Days\ 15 : 12 \end{matrix}\right\} :: 2100 : x$

$\therefore (6 \times 15 \times x)(9 \times 12 \times 2100)$

$\Leftrightarrow x = \left(\frac{9 \times 12 \times 2100}{6 \times 15}\right) = 2520.$

Hence, the required wages are Rs. 2520.

Example 4. If 20 men can build a wall 56 meters long in 6 days, what length of a similar wall can be built by 35 men in 3 days?

Solution.

Let the required length be x meters.

More men, More length built **(Direct Proportion)**

Less days, Less length built **(Direct Proportion)**

$\left.\begin{matrix} Men\ 20 : 35 \\ Days\ 6 : 3 \end{matrix}\right\} :: 56 : x$

$\therefore (20 \times 6 \times x) = (35 \times 3 \times 56)$

$x = \frac{(35 \times 3 \times 56)}{120} = 49.$

Hence, the required length is 49 m.

Example 5. If 15 men, working 9 hours a day, can reap a field in 16 days, in how many days will 18 men reap the field, working 8 hours a day?

Solution.

Let the required number of days be x.

More men, Less days **(Indirect Proportion)**

Less hours per day, More days **(Indirect Proportion)**

$$\left.\begin{matrix} Men & 18:15 \\ Hours\ per\ day & 8:9 \end{matrix}\right\} :: 16 : x$$

$\therefore (18 \times 8 \times x) = (15 \times 9 \times 16)$

$\Leftrightarrow x = \left(\frac{15 \times 144}{144}\right) = 15$

Hence, required number of days = 15.

Example 6. If engines consume 24 metric tonnes of coal, when each is working 8 hours a day, how much coal will be required for 8 engines, each running 13 hours a day, it being given that 3 engines of former type consume as much as 4 engines of latter type?

Solution.

Let 3 engines of former type consume 1 unit in 1 hour.

Then, 4 engines of latter type consume 1 unit in 1 hour.

$\therefore$ 1 engine of former type consumes $\frac{1}{3}$ unit in 1 hour.

1 engine of latter type consumes $\frac{1}{4}$ unit in 1 hour.

Let the required consumption of coal be x units.

Less engines, Less coal consumed **(Direct Proportion)**

More working hours, More coal consumed **(Direct Proportion)**

Less rate of consumption, Less coal consumed **(Direct Proportion)**

$$\left.\begin{matrix} Number\ of\ engines & 9:8 \\ Working\ hours & 8:13 \\ Rate\ of\ comsumption & \frac{1}{3}:\frac{1}{4} \end{matrix}\right\} :: 24 : x$$

$\therefore \left(9 \times 8 \times \frac{1}{3} \times x\right) = \left(8 \times 13 \times \frac{1}{4} \times 24\right)$

$\Leftrightarrow 24x = 624$

$\Leftrightarrow x = 26.$

Hence, the required consumption of coal = 26 metric tonnes.

Example 7. A contract is to be completed in 46 days and 117 men were set to work, each working 8 hours a day. After 33 days, $\frac{4}{7}$ of the work is completed. How many additional men may be employed so that the work may be completed in time, each man now working 9 hours a day?

Solution.

Remaining work $= \left(1 - \frac{4}{7}\right) = \frac{3}{7}$.

Remaining period $= (46 - 33) days = 13\ days$.

Let the total men working at it be x.

Less work, Less men **(Direct Proportion)**

Less days, More men **(Indirect Proportion)**

More Hrs/Day, Less men **(Indirect Proportion)**

$$\left.\begin{matrix} Work\ \frac{4}{7} : \frac{3}{7} \\ Days\ 13 : 33 \\ Hrs/Day\ \ 9 : 8 \end{matrix}\right\} :: 117 : x$$

$\therefore \frac{4}{7} \times 13 \times 9 \times x = \frac{3}{7} \times 33 \times 8 \times 117$ or

$x = \left(\frac{3 \times 33 \times 8 \times 117}{4 \times 13 \times 9}\right) = 198.$

$\therefore$ Additional men to be employed $= (198 - 117) = 81.$

Example 8. A garrison of 3300 men had provisions for 32 days, when given at the rate of 850 gms per head. At the end of 7 days, a reinforcement arrives and it was found that the provisions will last 17 days more, when given at the rate of 825 gms per head. What is the strength of the reinforcement?

Solution.

The problem becomes:

3300 men taking 850 gms per head have provisions for (32 – 7) or 25 days.

How many men taking 825 gms each have provisions for 17 days?

Less ratio per head, more men **(Indirect Proportion)**

Less days, More men **(Indirect Proportion)**

$$\left.\begin{matrix} Ratio\ \ 825 : 850 \\ Days\ \ 17 : 25 \end{matrix}\right\} :: 3300 : x$$

$\therefore 825\ \times 17 \times x = 850\ \times 25 \times 3300$ or

$x = \frac{850 \times 25 \times 3300}{825 \times 17} = 5000.$

$\therefore$ Strength of reinforcement $= (5500 - 3300) = 1700.$

Example 9. If the cost of x metres of wire is d rupees, then what is the cost of y meters of wire at the same rate?

Solution.

Cost of x meters $= Rs.d.Cost\ of\ 1\ meter$ = Rs. $\left(\frac{d}{x}\right)$.

Cost of y meters $= Rs.\left(\frac{d}{x} \times y\right) = Rs.\left(\frac{yd}{x}\right)$

Example 10. If a quarter kg of potato costs 60 paise, how many paise will 200 gm cost?

Solution.

Let the required cost be x paise.

Less weight, Less cost **(Direct Proportion)**

$\therefore 250 : 200 :: 60 : x$

$\Leftrightarrow 250 \times x = (200 \times 60)$

$\Leftrightarrow x = \frac{(200 \times 60)}{250}$

$\Leftrightarrow x = 48.$

Example 11. An industrial loom weaves 0.128 meters of cloth every second. Approximately, how many seconds will it take for the loom to weave 25 meters of cloth?

Solution.

Let the required time be x seconds.

Then,

More meters, more time **(Direct Proportion)**

$\therefore 0.128 : 25 :: 1 : x$

$\Leftrightarrow 0.128 \times x = 25 \times 1$

$\Leftrightarrow x = \frac{25}{0.128} = \frac{25 \times 1000}{128}$

$\Leftrightarrow x = 195.31$

$\therefore$ Required time $= 195\ sec$ (approximately).

Example 12. A flagstaff 17.5 m high casts a shadow of length 40.25 m. The height of the building, which casts a shadow of length 28.75 m under similar conditions will be:

Solution.

Let the height of the building be x meters.

Less lengthy shadow, Less is the height **(Direct Proportion)**

$\therefore 40.25 : 28.75 :: 17.5 : x$

$\Leftrightarrow 40.25 \times x = 28.75 \times 17.5$

$\Leftrightarrow x = \frac{(28.75 \times 17.5)}{40.25}$

$\Leftrightarrow x = 12.5$

Example 13. 36 men can complete a piece of work in 18 days. In how many days will 27 men complete the same work?

Solution.

Let the required number of days be x.

Then,

Less men, More days **(Indirect Proportion)**

$\therefore 27 : 36 :: 18 : x$

$\Leftrightarrow 27 \times x = 36 \times 18$

$\Leftrightarrow x = \frac{36 \times 18}{27}$

$\Leftrightarrow x = 24.$

Example 14. The cost of 16 packets of salt, each weighing 900 grams is Rs. 28. What will be the cost of 27 packets, if each packet weighs 1 kg?

Solution.

Let the required cost be Rs. x. Then,

More packets, More cost **(Direct Proportion)**

More weight, More cost **(Direct Proportion)**

$\left.\begin{matrix} Packets\ 16 : 27 \\ Weitht\ 900 : 1000 \end{matrix}\right\} :: 28 : x$

$\therefore (16 \times 900 \times x) = (27 \times 1000 \times 28)$

$\Leftrightarrow x = \frac{(27 \times 1000 \times 28)}{\mathbf{16} \times 900} = \frac{105}{2} = 52.50.$

Example 15. In a dairy farm, 40 cows eat 40 bags of husk in 40 days. In how many days one cow will eat one bag of husk?

Solution.

Let the required number of days be x.

Less cows, More days **(Indirect Proportion)**

Less bags, Less days **(Direct Proportion)**

$$\left.\begin{matrix} Cows & 1:40 \\ Bags & 40:1 \end{matrix}\right\} :: 40 : x$$

$$\therefore 1 \times 40 \times x = 40 \times 1 \times 40$$

$$\Leftrightarrow x = 40.$$

Example 16. 12 men working 8 hours per day complete a piece of work in 10 days. To complete the same work in 8 days, working 15 hours a day, the number of men required is:

Solution.

Let the required number of men be x.

Less days, more men **(Indirect Proportion)**

More working hrs per day, Less men **(Indirect Proportion)**

$$\left.\begin{matrix} Days & 8:10 \\ Working\ Hrs & 15:8 \end{matrix}\right\} :: 12 : x$$

$$\therefore 8 \times 15 \times x = 10 \times 8 \times 12$$

$$\Leftrightarrow x = \frac{10 \times 8 \times 12}{8 \times 15}$$

$$\Leftrightarrow x = 8.$$

Example 17. If 8 men can reap 80 hectares in 24 days, then how many hectares can 36 men reap in 30 days?

Solution.

Let the required number of hectares be x. Then,

More men, More hectares **(Direct Proportion)**

More days, More hectares **(Direct Proportion)**

$$\left.\begin{matrix} Men & 8:36 \\ Days & 24:30 \end{matrix}\right\} :: 80 : x$$

$$\therefore 8 \times 24 \times x = 36 \times 30 \times 80$$

$\Leftrightarrow x = \frac{(36\times30\times80)}{(8\times24)}$

$\Leftrightarrow x = 450.$

Example 18. 30 labourers, working 7 hours a day can finish a piece of work in 18 days. If the labourers work 6 hours a day then the number of labourers to finish the same piece of work in 30 days, will be:

Solution.

Let the required number of labourers be x. Then,

Less working hrs/day, More labourers **(Indirect Proportion)**

More days, Less labourers **(Indirect Proportion)**

$$\left.\begin{matrix} Working\ Hrs/Day & 6 : 7 \\ Days & 30 : 18 \end{matrix}\right\} :: 30 : x$$

$\therefore 6 \times 30 \times x = 7 \times 18 \times 30$

$\Leftrightarrow 6x = 126$

$\Leftrightarrow x = 21.$

Example 19. If 18 pumps can raise 2170 tonnes of water in 10 days, working 7 hours a day; in how many days will 16 pumps raise 1736 tonnes of water, working 9 hours a day?

Solution.

Let the required number of days be x. Then,

Less pumps, More days **(Indirect Proportion)**

Less weight, Less days **(Direct Proportion)**

More hours/ day, Less days **(Indirect Proportion)**

$$\left.\begin{matrix} Pumps & 16 : 18 \\ Weight & 2170 : 1736 \\ Hours/Day & 9 : 7 \end{matrix}\right\} :: 10 : x$$

$\therefore (16 \times 2170 \times 9 \times x) = (18 \times 1736 \times 7 \times 10)$

$\Leftrightarrow x = \frac{18\times1736\times7\times10}{16\times2170\times9} = 7.$

Example 20. If 12 carpenters, working 6 hours a day, can make 460 chairs in 24 days, how many chairs will 18 carpenters make in 36 days, each working 8 hours a day?

Solution.

Let the required number of chairs be x. Then,

More carpenters, More chairs **(Direct Proportion)**

More hours per day, More chairs **(Direct Proportion)**

More days, More chairs **(Direct Proportion)**

$$\left.\begin{matrix} Carpenters & 12:18 \\ Hours\ per\ day & 6:8 \\ Days & 24:36 \end{matrix}\right\} :: 460 : x$$

$\therefore (12 \times 6 \times 24 \times x) = (18 \times 8 \times 36 \times 460)$

$\Leftrightarrow x = \frac{(18\times8\times36\times460)}{(12\times6\times24)} = 1380.$

∴ Required number of chairs = 1380.

Example 21. 400 persons, working 9 hours per day complete $\frac{1}{4}th$ of the work in 10 days. The number of additional persons, working 8 hours per day, required to complete the remaining work in 20 days, is:

Solution.

Let the number of persons completing the work in 20 days be x.

Work done $= \frac{1}{4}$, Remaining work $= \left(1 - \frac{1}{4}\right) = \frac{3}{4}$.

Less hours per day, More men required **(Indirect Proportion)**

More work, More men required **(Direct Proportion)**

More days, Less men required **(Indirect Proportion)**

$$\left.\begin{matrix} Hours\ per\ day & 8:9 \\ Work & \frac{1}{4}:\frac{3}{4} \\ Days & 20:10 \end{matrix}\right\} :: 400 : x$$

$\therefore 8 \times \frac{1}{4} \times 20 \times x = 9 \times \frac{3}{4} \times 10 \times 400$

$\Leftrightarrow 40x = 27000$

$\Leftrightarrow x = 675.$

∴ Additional men $= (675 - 400) = 275.$

Example 22. If 18 binders bind 900 books in 10 days, how many binders will be required to bind 660 books in 12 days?

Solution.

Let the required number of binders be x.

Less books, Less binders **(Direct Proportion)**

More days, Less binders **(Indirect Proportion)**

$$\left.\begin{matrix} Books & 900 : 600 \\ Days & 12 : 10 \end{matrix}\right\} :: 18 : x$$

$\therefore (900 \times 12 \times x) = (600 \times 10 \times 18)$

$\Leftrightarrow x = \frac{600 \times 10 \times 18}{900 \times 12} = 11.$

Example 23. If $\frac{3}{5}$ of a cistern is filled in 1 minute, how much more time will be required to fill the rest of it?

Solution.

Let the required time be x seconds.

Part filled $= \frac{3}{5}$, Remaining part $= \left(1 - \frac{3}{5}\right) = \frac{2}{5}$.

Less part, Less time **(Direct Proportion)**

$\therefore \frac{3}{5} : \frac{2}{5} :: 60 : x$

$\Leftrightarrow \left(\frac{3}{5} \times x\right) = \left(\frac{2}{5} \times 60\right)$

$\Leftrightarrow x = 40.$

Example 24. If 5 engines consume 6 metric tonnes of coal when each is running 9 hours a day, how many metric tonnes of coal will be needed for 8 engines, each running 10 hours a day, it being given that 3 engines of the former type consume as much as 4 engines of the latter type?

Solution.

Let the required quantity of coal be x metric tonnes.

More engines, More coal **(Direct Proportion)**

More hours per day, More coal **(Direct Proportion)**

More rate, More coal **(Direct Proportion)**

$$\left.\begin{matrix} Engines & 5 : 8 \\ Hours\ per\ day & 9 : 10 \\ Rate & \frac{1}{3} : \frac{1}{4} \end{matrix}\right\} :: 6 : x$$

$\therefore \left(5 \times 9 \times \frac{1}{3} \times x\right) = \left(8 \times 10 \times \frac{1}{4} \times \frac{1}{6}\right)$

$\Leftrightarrow 15x = 120$

$\Leftrightarrow x = 8.$

Example 25. Some persons can do a piece of work in 12 days. Two times the number of such persons will do half of that work in:

Solution.

Let x men can do the work in 12 days and the required number of days be z.

More men, Less days **(Indirect Proportion)**

Less work, Less days **(Direct Proportion)**

$$\left.\begin{matrix} Men & 2x : x \\ Work & 1 : \frac{1}{2} \end{matrix}\right\} :: 12 : z$$

$\therefore (2x \times 1 \times z) = \left(x \times \frac{1}{2} \times 12\right)$

$\Leftrightarrow 2xz = 6x$

$\Leftrightarrow z = 3.$

Example 26. In a camp, 95 men had provisions for 200 days. After 5 days, 30 men left the camp. For how many days will the remaining food last now?

Solution.

Let the remaining food will last for x days.

95 men had provisions for 195 days.

65 men had provisions for x days.

Less men, More days **(Indirect Proportion)**

$\therefore 65 : 95 :: 195 : x$

$\Leftrightarrow (65 \times x) = (95 \times 195)$

$\Leftrightarrow x = \frac{95 \times 195}{65} = 285.$

Example 27. 15 men take 21 days of 8 hours each to do a piece of work. How many days of 6 hours each would 21 women take, if 3 women do as much work as 2 men?

Solution.

3 women ≡ 2 men. So, 21 women ≡14 men.

Less men, More days **(Indirect Proportion)**

Less hours per day, More days **(Indirect Proportion)**

$$\left.\begin{matrix} Mens & 14:15 \\ Hours\ per\ day & 6:8 \end{matrix}\right\} :: 21 : x$$

$\therefore (14 \times 6 \times x) = (15 \times 8 \times 21)$

$\Leftrightarrow x = \frac{(15\times8\times21)}{(14\times6)} = 30.$

∴ Required number of days = 30.

Example 28. A contractor undertook to do a certain piece of work in 9 days. He employed certain number of men, but 6 of them being absent from the very first day, the rest could finish the work in 15 days. The number of men originally employed were:

Solution.

Let there be x men at the beginning.

Less men, More days **(Indirect Proportion)**

$\therefore 15 : 9 :: x : (x - 6)$

$\Leftrightarrow 15\,(x - 6) = 9x$

$\Leftrightarrow 6x = 90$

$\Leftrightarrow x = 15.$

Example 29. 12 men and 18 boys , working $7\frac{1}{2}$ hours a day, can do a piece of work in 60 days. If a man works equal to 2 boys, then how many boys will be required to help 21 men to do twice the work in 50 days, working 9 hours a day?

Solution.

$1\ man \equiv 2\ boys$

$\Leftrightarrow (12\ men + 18\ boys) \equiv (12 \times 2 + 18) boys \equiv 42\ boys.$

Let required number of boys $\equiv x.\ 21\ men + x\ boys \equiv (21 \times 2 + x) boys$

$= (42 + x) boys.$

Less days, More boys **(Indirect Proportion)**

More hrs per day, Less boys **(Indirect Proportion)**

$$\left.\begin{array}{ll} Days & 50:60 \\ Hours\ per\ day & 9:\frac{15}{2} \\ Work & 1:2 \end{array}\right\} :: 42:(42+x)$$

$\therefore [50 \times 9 \times 1 \times (42+x)] = \left(60 \times \frac{15}{2} \times 2 \times 42\right)$

$\Leftrightarrow (42+x) = \frac{37800}{450}$

$\Leftrightarrow 42 + x = 84$

$\Leftrightarrow x = 42.$

Example 30. If 3 men or 6 boys can do a piece of work in 10 days, working 7 hours a day; how many days will it take to compete a piece of work twice as large with 6 men and 2 boys working together for 8 hours a day?

Solution.

$3\ men \equiv 6\ boys$

$\Leftrightarrow (6\ men + 2\ boys) \equiv 14\ boys.$

More work, More days **(Direct Proportion)**

More boys, Less days **(Indirect Proportion)**

More hours per day, Less days **(Indirect Proportion)**

$$\left.\begin{array}{ll} Work & 1:2 \\ Boys & 14:6 \\ Hours\ per\ day & 8:7 \end{array}\right\} :: 10:x$$

$\therefore (1 \times 14 \times 8 \times x) = (2 \times 6 \times 7 \times 10)$

$\Leftrightarrow x = \frac{840}{112} = 7\frac{1}{2}.$

EXERCISE

1. If the price of 6 toys is Rs. 264.37, what will be the approximate price of 5 toys?
2. The price of 357mangoes is Rs. 1517.25. what will be the approximate price of 9 dozens of such mangoes?
3. If 11.25 m of a uniform iron rod weighs 42.75 kg, what will be the weight of 6 m of the same rod?
4. On a scale of map, 0.6 cm represents 6.6 km. If the distance between the points on the map is 80.5 cm, the actual distance between these points is:
5. A man completes $\frac{5}{8}$ of a job in 10 days. At this rate, how many more days will it take him to finish the job?

6. A fort had provision of food for 150 men for 45 days. After 10 days, 25 men left the fort. The number of days for which the remaining food will last, is:
7. A wheel that has 6 cogs is meshed with a larger wheel of 14 cogs. When the smaller wheel has made 21 revolutions, then the number of revolutions made by the larger wheel is:
8. In a camp, there is a meal for 120 mentor 200 children. If 150 children have taken the meal, how many men will be catered to with the remaining meal?
9. 4 mat-weavers can weave 4 mats in 4 days. At the same rate, how many mats would be woven by 8 mat-weavers in 8 days?
10. Running at the same constant rate, 6 identical machines can produce a total of 270 bottles per minute. At this rate, how many bottles could 10 such machines produce in 4 minutes?
11. 10 men, working 6 hours a day can complete a work in 18 days. How many hours a day must 15 men work to complete the same work in 12 days?
12. 39 persons can repair a road in 12 days, working 5 hours a day. In how many days will 30 persons, working 6 hours a day, complete the work?
13. 3 pumps, working 8 hours a day, can empty a tank in 2 days. How many hours a day must 4 pumps work to empty the tank in 1 day?
14. A certain number of persons can dig a trench 100 m long, 50 m broad and 10 m deep in 10 days. The same number of persons can dig another trench 20 m broad and 15 m deep in 30 days. The length of the second trench is:
15. If 5 men or 9 women can do a piece of work in 19days, then in how many days will 3 men and 6 women do the same work?
16. 49 pumps can empty a reservoir in $6\frac{1}{2}$ days, working 8 hours a day. If 196 pumps are used for 5 hours each day, then the same work will be completed in:
17. If 7 spiders make 7 webs in 7 days, then 1 spider will make 1 web in how many days?
18. If 80 lamps can be lighted, 5 hours per day for 10 days for Rs. 21.25, then the number of lamps, which can be lighted 4 hours daily for 30 days, for Rs. 76.50, is:
19. If 9 examiners can examine a certain number of answer books in 12 days, working 5 hours a day; for how many hours a day would 4 examiners have to work in order to examine twice the number of answer books in 30 days?
20. If 17 labourers can dig a ditch 20m long in 18 days, working 8 hours a day; how many more labourers should be engaged to dig a similar ditch 39 m long in 6 days, each labourer working 9 hours a day?
21. 20 men complete one-third of a piece of work in 20 days. How many more men should be employed to finish the rest of the work in 25 more days?
22. If x men, working x hours per day, can do x units of work in x days, then y men, working y hours per day would be able to complete how many units of work in y days?
23. A rope makes 70 rounds of the circumference of a cylinder whose radius of the base is 14 cm. How many times can it go round a cylinder with radius 20 cm?
24. If a certain number of workmen can do a piece of work in 25 hours, in how many hours will another set of an equal number of men, do a piece of work, twice as great, supposing that 2 men of the first set can do as much work in an hour, as 3 men of the second set do in an hour?

25. A certain number of men can finish a piece of work in 100 days. If , there were 10 men less, it would take 10 days more for the work to be finished. How many men were there originally?

26. A garrison of 500 men had provisions for 27 days. After 3 days a reinforcement of 300 men arrived. For how many more days will the remaining food last now?

27. A garrison had provisions for a certain number of days. After 10 days, $\frac{1}{5}$ of the men desert and it is found that the provisions will now last just as long as before. How long was that?

28. A contractor undertake to do a piece of work in 40 days. He engages 100 men at the beginning and 100 more after 35 days and completes the work in stipulated time. If he had not engaged the additional men, how many days behind schedule would it be finished?

29. A contractor employed 30 men to do a piece of work in 38 days. After 25 days, he employed 5 men more and the work was finished one day earlier. How many days he would have been behind, if he had not employed additional men?

30. 2 men and 7 boys can do a piece of work in 14 days; 3 men and 8 boys can do the same in 11 days. Then, 8 men and 6 boys can do three times the amount of this work in:

ANSWERS

1. Rs. 220	2. Rs. 2500	3. x = 22.8	4. x= 885.5
5. x = 6	6. 42 days	7. x =9	8. 3 men
9. x = 16	10. x = 1800	11. x = 6	12. x = 13
13. x = 12	14. x = 500	15. x = 15	16. $x = 2\frac{3}{5}$
17. x =7	18. x =120	19. x = 9	20. 51
21. 12	22. $z = \frac{y^3}{x^2}$	23. x = 49	24. x = 75
25. x = 110	26. x = 15	27. y = 50	28. 5 days
29. 1 day	30. 21 days		

UNIT V

PIPES AND CISTERNS

INTRODUCTION

I. **Inlet:** A pipe connected with a tank or a cistern or a reservoir, that fills it, is known as an inlet

Outlet: A pipe connected with a tank or a cistern or a reservoir, emptying it, is known as an outlet.

II. (i) If a pipe can fill a tank in x hours, then part filled in 1 hour = $\frac{1}{x}$.

(ii) If a pipe can empty a full tank in y hours, then part emptied in 1 hour = $\frac{1}{y}$.

(iii) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where $y > x$), then on opening both the pipes, the net part filled in 1 hour = $\left(\frac{1}{x} - \frac{1}{y}\right)$

(iv) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where x>y), then on opening both the pipes, the net part emptied in 1 hour = $\left(\frac{1}{y} - \frac{1}{x}\right)$.

SOLVED PROBLEMS

Example 1. A tank 9 ft by 5 ft by 2ft is fitted with an inlet pipe and an exhaust pipe. The inlet pipe pours in 576 cu. inch of water per minute and the exhaust pipe can empty the full tank in 3 hours. If the tank is full and both pipes are open, how many hours will it take to empty it?

Solution.

Volume of the tank $= (9 \times 5 \times 2)\ cu.ft = 90\ cu.ft.$

$= (9 \times 12 \times 12 \times 12)\ cu.inch$

Volume of water drained by the exhaust pipe in one minute

$= \left(\frac{90 \times 12 \times 12 \times 12}{3 \times 60}\right)$ cu. inch = 864 cu. Inch.

Net Volume drained in one minute, when both the pipes are opened

$= (864 - 576)\ cu.inch$ = 288 cu. Inch

∴ Required time $= \left(\frac{90 \times 12 \times 12 \times 12}{28860}\right)$ hrs = 9 hrs

Example 2. Pipe A can fill a tank in 30 hours and pipe B in 45 hours. If both the pipes are opened in an empty tank, how much time will they take to fill it?

Solution.

Part filled by A in 1 hour = $\frac{1}{30}$

Part filled by B in 1 hour = $\frac{1}{45}$

Part filled by (A+B) in 1 hour = $\left(\frac{1}{30}+\frac{1}{45}\right)=\frac{10}{180}=\frac{1}{18}$

Hence, pipes A and B together will fill the tank in 18 hours.

Example 3. A cistern can be filled by pipes A and B in 4 hours and 6 hours respectively. When full, the cistern can be emptied by pipe C in 8 hours. If all the pipes were turned on at the same time, in how much time will the cistern be filled?

Solution.

Net part filled in 1hour = $\left(\frac{1}{4}+\frac{1}{6}-\frac{1}{8}\right)=\frac{7}{24}$

∴ The cistern will be full in $\frac{24}{7}$ hrs

i.e., $3\frac{3}{7}$ hrs.

Example 4. A pipe can empty a tank in 40 minutes. A second pipe with diameter twice as much as that of the first is also attached with the tank to empty it. How much time will the two pipes together take to empty the tank?

Solution.

Let the diameters of the pipes be d and 2d and the times taken by them to empty the tank be t and T minutes respectively.

Since the time taken to fill or empty a tank is inversely proportional to the square of the diameter of the pipe, we have:

$$t \propto \frac{1}{d^2} \Rightarrow t=\frac{k}{d^2} \Rightarrow \frac{k}{d^2}=40$$

Again, $T \propto \frac{1}{(2d)^2} \Rightarrow T=\frac{k}{4d^2} \Rightarrow T=\frac{1}{4}\times\frac{k}{d^2}=\frac{1}{4}\times 40=10$

Thus, net part emptied in 1 min = $\left(\frac{1}{40}+\frac{1}{10}\right)=\frac{5}{40}=\frac{1}{8}$

Hence, the two pipes together will empty the tank in 8 minutes.

Example 5. A tap can fill a tank in 10 minutes and another can empty it in 6 minutes. If the tank is already two-fifths full and both the taps are opened together, will the tank be filled or emptied? How long will it take before the tank is either filled completely or emptied completely, as the case may be?

Solution.

Clearly, the outlet pipe is faster than the inlet pipe and so, the tank will be emptied.

Part to be emptied $=\frac{2}{5}$

Net part emptied in 1 minute $= \left(\frac{1}{6} = \frac{1}{10}\right) = \frac{2}{30} = \frac{1}{15}$

$\therefore \frac{1}{15} : \frac{2}{5} :: 1 : x$ or $x = \left(\frac{2}{5} \times 1 \times 15\right) =$ min

So, the tank will be emptied in 6 minutes.

Example 6. A cistern has two taps which fill it in 12 minutes and 15 minutes respectively. There is also a waste pipe in the cistern. When all the three are opened, the empty cistern is full in 20 minutes. How long will the waste pipe take to empty the full cistern?

Solution.

Work done by the waste pipe in 1minute

$$= \frac{1}{20} - \left(\frac{1}{12} + \frac{1}{15}\right) = -\frac{1}{10}$$

∴ Waste pipe will empty the full cistern in 10 minutes.

Example 7. An electric pump can fill a tank in 3 hours. Because of a leak in the tank it took $3\frac{1}{2}$ hours to fill the tank. If the tank is full, how much time will the leak take to empty it?

Solution.

Work done by the leak in 1hour $= \left[\frac{1}{3} - \frac{1}{\left(\frac{7}{2}\right)}\right] = \left(\frac{1}{3} - \frac{2}{7}\right) = \frac{1}{21}$

∴ The leak will empty the tank in 21 hours.

Example 8. Two pipes can fill a cistern in 14 hours and 16 hours respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom it took 32 minutes more to fill the cistern. When the cistern is full, in what time will the leak empty it?

Solution.

Work done by the two pipes in 1 hour$= \left(\frac{1}{14} + \frac{1}{16}\right) = \frac{15}{112}$

∴ Time taken by these pipes to fill the tank $= \frac{112}{15}\ hrs$

$= 7\ hrs\ 28\ min$

Due to leakage, time taken $= 7\ hrs\ 28\ min + 32\ min = 8\ hrs.$

∴ Work done by (two pipes + leak) in 1 hour $= \frac{1}{8}$

Work done by the leak in 1 hour $= \left(\frac{15}{112} - \frac{1}{8}\right) = \frac{1}{112}$

Hence, the leak will empty the full cistern in 112 hours.

Example 9. If two pipes function simultaneously, the reservoir will be filled in 12 hours. The second pipe fills the reservoir 10 hours faster than the first. How many hours does it take the second pipe to fill the reservoir?

Solution.

Let the reservoir be filled by first pipe in x hours.

Then, second pipe will fill it in $(x + 10)$ hours.

$\therefore \frac{1}{x} + \frac{1}{(x+10)} = \frac{1}{12} \Leftrightarrow \frac{x+10+x}{x(x+10)} = \frac{1}{12}$

$\Leftrightarrow x^2 - 14x - 120 = 0$

$\Leftrightarrow (x - 20)(x + 6) = 0$

$\Leftrightarrow x = 20$

So, the second pipe will take $(20 + 10)$

i.e. 30 hrs to fill the reservoir.

Example 10. A tank is fitted with two inlet pipes A and B, and an outlet pipe C. A is twice as efficient as B which in turn is twice as efficient as C. The empty tank gets filled in 16 hours when all the three pipes are opened. How many hours will be taken to fill the empty tank if B is plugged and the other two pipes are opened?

Solution.

Suppose A alone takes x hours to fill the tank. Then, B alone takes 2x hours to fill the tank and C alone takes 4x hours to empty the tank.

[Q time taken to fill or empty a tank is inversely proportional to the efficiency of the pipe]

$\therefore \frac{1}{x} + \frac{1}{2x} - \frac{1}{4x} = \frac{1}{16} \Leftrightarrow \frac{8+4-2}{8x} = \frac{1}{16}$

$\Leftrightarrow x = \left(\frac{10\times16}{8}\right) = 20$

Net part filled by A and C in 1 hour $= \left(\frac{1}{x} - \frac{1}{4x}\right) = \frac{3}{4x} = \frac{3}{80}$

Hence, A and C will fill the tank in $\frac{80}{3}$ hrs.

i.e. $26\frac{2}{3}$ hrs. (or) 26 hrs 40 mins.

Example 11. Two pipes A and B can fill a tank in 12 minutes and 15 minutes respectively while a third pipe C can empty the full tank in 20 minutes. All the three pipes are opened in the beginning. However, pipe C is closed 6 minutes before the tank is filled. In what time will the tank be full?

Solution.

Let the tank be full in x minutes.

Then, pipes A and B worked for x minutes, while pipe C worked for $(x-6)$ minutes.

$$\therefore \frac{x}{12} + \frac{x}{15} - \frac{(x-6)}{20} = 1 \Leftrightarrow \frac{5x+4x-3(x-6)}{60} = 1$$

$$\Leftrightarrow 6x + 18 = 60$$

$$\Leftrightarrow 6x = 42$$

$$\Leftrightarrow x = 7.$$

Hence, the tank will be full in 7 minutes.

Example 12. Pipes A and B can completely fill a water tank in 4 hours and 5 hours respectively. A pipe C can empty a tank filled completely with water in 3 hours. Initially, the tank is empty and all pipes are closed. Pipe A is opened first at time $t = 0$ and pipe C is opened at the instant when the tank is exactly half filled with water. Pipe B is opened after pipe C and at the instant when the tank is exactly one-fourth filled with water. Find the total time taken to fill the tank completely counting from $t = 0$.

Solution.

The whole process involves 3 steps:

Step 1: Pipe A alone fills half the tank.

Let time taken t_1. Then, $t_1 = 2$ hours.

Step 2: Pipe A and C together empty $\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{1}{4}$ of the tank.

Let time taken be t_2.

Work alone by A and C in 1 hour $\left(\frac{1}{4} - \frac{1}{3}\right) = -\frac{1}{12}$

$\therefore \frac{1}{12} : \frac{1}{4} :: 1 : t_2$ **(or)** $t_2 = \frac{1}{4} \times 12 = 3\ hrs.$

Step 3: Pipes A, B and C together fill$\left(1-\frac{1}{4}\right)=\frac{3}{4}$ of the tank.

Let time taken be t_3.

Works done by A, B and C in 1 hour $=\left(\frac{1}{4}+\frac{1}{5}-\frac{1}{3}\right)=\frac{7}{60}$.

$\therefore \frac{7}{60}:\frac{3}{4}::1:t_3$ **(or)** $t_3=\frac{3}{4}\times\frac{60}{7}=\frac{45}{7}=6\frac{3}{7}$ hrs.

Hence, total time taken $=(t_1+t_2+t_3)=\left(2+3+6\frac{3}{7}\right)$ hrs.

Example 13. A cistern has three pipes A, B and C. A and B can fill it in 3 hours and 4 hours respectively while C can empty the completely filled cistern in 1 hour. If the pipes are opened in order at 3, 4 and 5 p.m. respectively, at what time will the cistern be empty?

Solution.

Let the cistern be emptied t hours after 3 p.m.

Then, work done by pipe A in t hour + work done by pipe B in $(t-1)$ hours + work done by pipe C in $(t-2)$ hours $=0$.

$\Rightarrow \frac{t}{3}+\frac{(t-1)}{4}-\frac{(t-2)}{1}=0$

$\Rightarrow 4t+3(t-1)-12(t-2)=0$

$\Rightarrow -5t+21=0$

$\Rightarrow 5t=21$

$\Rightarrow t=4\frac{1}{5}\ hrs=4\ hrs\ 12\ min$

So, the cistern will be emptied 4 hrs 12 min after 3 p.m.

i.e., 7 : 12 p.m.

Example 14. Three pipes A, B and C are attached to a tank. A and B can fill it in 20 and 30 minutes respectively while C can empty it in 15 minutes. If A, B and C are kept open successively for 1 minute each, how soon will the tank be filled?

Solution.

$(A\&+\&B\&+\&C)'s$ 3 minutes work when opened alternately $=\left(\frac{1}{20}+\frac{1}{30}-\frac{1}{15}\right)=\frac{1}{60}$

Part filled in $(3\&\times\&55)$ i.e., 165 min $=\frac{55}{60}=\frac{11}{12}$

Remaining part $=\left(1-\frac{11}{12}\right)=\frac{1}{12}$

Now it is A′s turn.

Part filled by A in 1 min $= \frac{1}{20}$

Remaining part $= \left(\frac{1}{12} - \frac{1}{20}\right) = \frac{1}{30}$, which is filled by B in the next minute

So, total time taken $= (165 + 2)$ min $= 167$ min $= 2$ hrs 47 min.

Example 15. Two pipes A and B can fill a tank in 24 minutes and 32 minutes respectively. If both the pipes are opened simultaneously, after how much time B should be closed so that the tank is full in 18 minutes?

Solution.

Let B be closed after x minutes.

Then, Part filled by $(A + B)$ in x min + part filled by A in $(18 - x)$ min $= 1$

$\therefore x\left(\frac{1}{24} + \frac{1}{32}\right) + (18 - x) \times \frac{1}{24} = 1$

$\Rightarrow \frac{7x}{96} + \frac{18-x}{24} = 1$

$\Rightarrow 7x + 4(18 - x) = 96$

$\Rightarrow x = 8$

Hence, B must be closed after 8 minutes.

Example 16. A keg is fitted with 3 taps – A, B and C. All the three taps, if opened together, can drain the full keg in $1\frac{1}{2}$ minutes. Taps B and C together take 2 minutes to drain the keg while taps A and C together take $2\frac{4}{13}$ minutes to drain it. How long will taps A and B together take to drain the keg?

Solution.

Let taps A, B and C individually take, x, y and z minutes respectively to drain the keg.

Then, $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{2}{3}; \frac{1}{y} + \frac{1}{z} = \frac{1}{2}; \frac{1}{x} + \frac{1}{z} = \frac{13}{30}$

$(A + B)$'s 1 minute's work $= \frac{1}{x} + \frac{1}{y}$

$= 2\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) - \left(\frac{1}{y} + \frac{1}{z}\right) - \left(\frac{1}{x} + \frac{1}{z}\right)$

$= 2 \times \frac{2}{3} - \frac{1}{2} - \frac{13}{30}$

$= \frac{4}{3} - \frac{1}{2} - \frac{13}{30}$

$= \frac{12}{30}$

$= \frac{2}{5}$

Hence, A and B together can drain the keg in $\frac{5}{2}$

i.e., $2\frac{1}{2}$ minutes.

Example 17. The petrol tank of an automobile can hold g litres. If a litres was removed when the tank was full, what part of the full tank was removed?

Solution.

Required part = $\frac{Quantity\ removed}{Total\ capacity} = \frac{a}{g}$

Example 18. A tap can completely fill a water tank in 8 hours. The water tank has a hole in it through which the water leaks out. The leakage will cause the full water tank to get empty in 12 hours. How much time will it take for the tap to fill the tank completely with the hole?

Solution.

Net part filled in 1hour =$\left(\frac{1}{8} - \frac{1}{12}\right) = \frac{1}{24}$

∴ The tank will be filled in 24 hours.

Example 19. A tap can fill a tank in 6 hours. After half the tank is filled, three more similar taps are opened. What is the total time taken to fill the tank completely?

Solution.

Time taken by one tap to fill half the tank = 3 hrs.

Part filled by the four taps in one hour = $\left(4 \times \frac{1}{6}\right) = \frac{2}{3}$

Remaining Part = $\frac{1}{2}$

$\therefore \frac{3}{40} : \frac{3}{4} :: 1 : x$

$x = \left(\frac{3}{4} \times 1 \times \frac{40}{3}\right) = 10\ hrs$

So, the cistern will be emptied in 10 hours.

Example 20. Three pipes A, B and C can fill a tank from empty to full in 30 minutes, 20 minutes and 10 minutes respectively. When the tank is empty, all the three pipes are opened. A, B and C discharge chemical solutions P, Q and R respectively. What is the proportion of solution R in the liquid in the tank after 3 minutes?

Solution.

Part filled by $(A+B+C)$ in 3 minutes $= 3\left(\frac{1}{30}+\frac{1}{20}+\frac{1}{10}\right)$

$$= \left(3 \times \frac{11}{60}\right)$$

$$= \frac{11}{20}$$

Part filled by C in 3 minutes $= \frac{3}{10}$

$\therefore$ Required ratio $= \left(\frac{3}{10} \times \frac{20}{11}\right) = \frac{6}{11}$

Example 21. A tank is filled in 5 hours by three pipes A, B and C. The pipe C is twice as fast as B and B is twice as fast as A. How much time will pipe A alone take to fill the tank?

Solution.

Suppose pipe A alone takes x hours to fill the tank.

Then, pipes B and C will take $\frac{x}{2}$ and $\frac{x}{4}$ hours respectively to fill the tank.

$\therefore \frac{1}{x}+\frac{2}{x}+\frac{4}{x}=\frac{1}{5}$

$\Rightarrow \frac{7}{x}=\frac{1}{5}$

$\Rightarrow x = 35$

So, pipe A alone takes 35 hours to fill the tank.

Example 22. A swimming pool is filled by three pipes with uniform flow. The first two pipes operating simultaneously fill the pool in the same time during which the pool is filled by the third pipe alone. The second pipe fills the pool 5 hours faster than the first pipe and 4 hours slower than the third pipe. The time required by the first pipe is

Solution.

Suppose first pipe alone takesx hours to fill the tank.

Then, second and third pipes will take $(x-5)$ and $(x-9)$ hours respectively to fillthe tank.

$\therefore \frac{1}{x}+\frac{1}{(x-5)}=\frac{1}{(x-9)} \Leftrightarrow \frac{x-5+x}{x(x-5)}=\frac{1}{(x-9)}$

$$\Leftrightarrow (2x-5)(x-9) = x(x-5)$$

$$\Leftrightarrow x^2 - 18x + 45 = 0$$

$$\Leftrightarrow (x-15)(x-3) = 0$$

$$\Leftrightarrow x = 15\ [neglecting\ x = 3]$$

So, first pipe alone takes 15 hrs to fill the tank.

Example 23. Two pipes A and B can fill a tank in 12 minutes and 15 minutes respectively. If both the pipes are opened simultaneously and pipe A is closed after 3 minutes, then how much more time will it take to fill the tank by pipe B?

Solution.

Part filled in 3 min $= 3\left(\frac{1}{12}+\frac{1}{15}\right) = \left(3\times\frac{9}{60}\right) = \frac{9}{20}$

Remaining part $= \left(1-\frac{9}{20}\right) = \frac{11}{20}$

Part filled by B in 1 min $\frac{1}{15}$

$\frac{1}{15}:\frac{11}{20} :: 1: x$

$\Rightarrow x = \left(\frac{11}{20}\times 1\times 15\right)$

$= 8\frac{1}{4}\,\text{min}$

$= 8\,\text{min}\,15\,sec$

∴ Remaining part is filled by B in 8 min 15 sec.

Example 24. Tap A fills a tank in 4 hours whereas tap B empties the full tank in 24 hours. A and B are opened alternately for 1hour each. Every 2 hours the level of water is found to increase by 0.5 m. The depth of the tank is

Solution.

Part filled in 2 hours $=\left(\frac{1}{4}-\frac{1}{24}\right) = \frac{5}{24}$

Let the depth of the tank be h metres.

Then, $\frac{5}{24}h = 0.5$

$\Rightarrow \quad h = \left(\frac{0.5\times 24}{5}\right) = 2.4\,min$

Example 25. Two pipes can fill a tank in 20 and 24 minutes respectively and a waste pipe can empty 3 gallons per minute. All the three pipes working together can fill the tank in 15 minutes. The capacity of the tank is:

Solution.

Work done by the waste pipe in 1 minute

$$= \frac{1}{15} - \left(\frac{1}{20} + \frac{1}{24}\right) = \left(\frac{1}{15} - \frac{11}{120}\right) = -\frac{1}{40}$$ [-ve sign means emptying]

∴ Volume of $\frac{1}{40}$ part = 3 gallons.

Volume of whole tank = (3×40) gallons =120 gallons

Example 26. Three pipes can fill a reservoir in 10, 15 and 20 hours respectively. If the three taps are opened one after another in the given order, with a certain fixed time gap between them, the reservoir fills in 5 hours. The time gap is

Solution.

Let the fixed time gap be x hrs. Then, part filled by first pipe in 5 hrs + part filled by second pipe in (5 –x) hrs + part filled by third pipe in (5-2x) hrs = 1

$$\Longrightarrow \frac{5}{10} + \frac{(5-x)}{15} + \frac{(5-2x)}{20} = 1$$

$$\Longrightarrow 30 + 4(5 - x) + 3(5 - 2x) = 60$$

$$\Longrightarrow 10x = 5$$

$$\Longrightarrow x = \frac{1}{2}.$$

Hence, the fixed time gap is $\frac{1}{2}hr$ i.e. 30 min

Example 27. A tank is 7 metre long and 4 metre wide. At what speed should water run through a pipe 5 cm broad and 4 cm deep so that in 6 hours and 18 minutes water level in the tank rise by 4.5 metre?

Solution:

Rate of flow of water $= x\ cm/minute$

∴ Volume of water that flowed in the tank in 1 minutes $= 5 \times 4 \times x = 20x\ cu.cm.$

∴ Volume of water that flowed in the tank in 6 hours 18 minutes.

i.e. $(6 \times 60 + 18) = 378\ minutes$

$$= 2x \times 378\ cu.cm$$

According to the question, $20x \times 378 = 700 \times 400 \times 450$

$$\Longrightarrow x = \left(\frac{700\times400\times450}{20\times378}\right)\ cm/minutes$$

$$= \left(\frac{700\times400\times450\times60}{100000\times20\times378}\right)\ km/hour.$$

$$= 10\ km/hour.$$

Example 28. Two pipes can fill a tank in 12 hours and 16 hours respectively. A third pipe can empty the tank in 30 hours. If all the three pipes are opened and function simultaneously, they in how much time the tank will be full?(in hours)

Solution.

First pipe fill the tank in 1 hour $= \frac{1}{12}$ part of tank

Second pipe fill the tank in 1 hour $= \frac{1}{16}$ part of tank

Third pipe empty the tank in 1 hour $= \frac{1}{30}$ part of tank.

When all three pipes are opened simultaneously, part of the tank filled in 1 hour

$= \frac{1}{12} + \frac{1}{16} - \frac{1}{30}$

LCM of $12,16\ and\ 30 = 240$

$= \frac{20+15-8}{240} = \frac{27}{240}$

∴ Required time taken by all the three pipes $= \frac{240}{27}$

$= \frac{80}{9} = 8\frac{8}{9}$ hours.

EXERCISE

1. In 1 minute, $\frac{3}{7}$ of a bucket is filled. The rest of the bucket can be filled in
2. Water is continuously supplied from a reservoir to a locality at the steady rate of 10,000 litres per hour. When delivery exceeds demand the excess water is stored in a tank. If the demand for 8 consecutive three-hour periods is 10000, 10000, 45000, 25000, 40000, 15000, 60000 and 35000 litres respectively, what will be the minimum capacity required of the water tank (in thousand litres) to meet the demand and avoid any wastage?
3. Two pipes A and B can fill a tank in 20 and 30 minutes respectively. If both the pipes are used together, how long will it take to fill the tank?
4. A pipe can fill a tank in x hours and another pipe can empty it in y $(y > x)$ hours. If both the pipes are open, in how many hours will the tank be filled?
5. A tank with capacity T litres is empty. If water flows into the tank from pipe X at the rate of x litres per minute and water is pumped out by Y at the rate of y litres per minute and $x > y$, then in how many minutes will the tank be filled?
6. Pipes A and B can fill a tank in 20 hours and 30 hours respectively and pipe C can empty the full tank in 40 hours. If all the pipes are opened together, how much time will be needed to make the tank full?
7. A pipe can fill a tank in 3 hours. There are two outlet pipes from the tank which can empty it in 7 and 10 hours respectively. If all the three pipes are opened simultaneously, then the tank will be filled in

8. In what time would a cistern be filled by three pipes whose diameters are 1 cm, $1\frac{1}{3}$cm and 2 cm running together, when the largest alone will fill it in 61 minutes, the amount of water flowing in by each pipe, being proportional to the square of its diameter?
9. A cistern has two pipes. One can fill it with water in 8 hours and other can empty it in 5 hours. In how many hours will the cistern be emptied if both the pipes are opened together when $\frac{3}{4}$ of the cistern is already full of water?
10. A vessel has three pipes connected to it, two to supply liquid and one to draw liquid. The first alone can fill the vessel in $4\frac{1}{2}$ hours, the second in 3 hours and the third can empty it in $1\frac{1}{2}$ hours. If all the pipes are opened simultaneously when the vessel is half full, how soon will it be emptied?
11. Two pipes A and B can separately fill a cistern in 60 minutes respectively. There is a third pipe in the bottom of the cistern to empty it. If all the three pipes are simultaneously opened then the cistern is full in 50 minutes. In how much time, the third pipe alone can empty the cistern?
12. Eight pipes are fitted to a water tank. Some of these are water pipes to fill the rank and the remaining are waste pipes used to empty the tank. Each water pipe can fill the tank in 12 hours and each waste pipe can empty it in 36 hours. On opening all the pipes an empty tank is filled in 3 hours. The number of waste pipes is
13. A pump can fill a tank with water in 2 hours. Because of a leak, it took $2\frac{1}{3}$ hours to fill the tank. The leak can drain all the water of the tank in.
14. Two taps A and B can fill a tank in 5 hours and 20 hours respectively. If both the taps are open then due to a leakage, it took 30 minutes more to fill the tank. If the tank is full, how long will it take for the leakage alone to empty the tank?
15. Two pipes A and B together can fill a cistern in 4 hours. Had they been opened separately, then B would have taken 6 hours more than A to fill the cistern. How much time will be taken by A alone to fill the cistern?
16. One pipe can fill a tank three times as fast as another pipe. If together the two pipes can fill the tank in 36 minutes, then the slower pipe alone will be able to fill the tank in
17. 12 buckets of water fill a tank when the capacity of each bucket is 13.5 litres. How many buckets will be needed to fill the same tank, if the capacity of each bucket is 9 litres?
18. Bucket P has thrice the capacity as bucket Q. It takes 60 turns for bucket P to fill the empty drum. How many turns will it take for both the buckets P and Q, having each turn together to fill the empty drum?
19. Two pipes A and B can fill a tank in 15 minutes and 20 minutes respectively. Both the pipes are opened together but after 4 minutes, pipe A is turned off. What is the total time required to fill the tank?
20. Two pipes A and B can fill a tank in 15 hours and 20 hours respectively while a third pipe C can empty the full tank in 25 hours. All the three pipes are opened in the beginning. After 10 hours, C is closed. In how much time will the tank be full?
21. A large tanker can be filled by two pipes A and B in 60 minutes and 40 minutes respectively. How many minutes will it take to fill the tanker from empty state if B is used for half the time and A and B fill it together for the other half?

22. Two pipes A and B can fill a cistern in 12 minutes and 15 minutes respectively while a third pipe C can empty the full cistern in 6 minute. A and B are kept open for 5 minutes in the beginning and then C is also opened. In what time is the cistern emptied?

23. Pipe A can fill a tank in 10 hours. Pipe B can fill the same tank in 15 hours. Pipe C can empty the full tank in 20 hours. Pipes A, B and C are opened alternatively for one hour each. If A is opened first, then how many hours will they take to fill the empty tank?

ANSWERS

1. $\frac{4}{3}$ min
2. 40
3. 12min
4. $\left(\frac{xy}{y-x}\right)$ hours
5. $\frac{T}{(x-y)}$ min
6. $17\frac{1}{7}$ hours
7. $\cong$ 11 hrs
8. 36 min
9. 10 hrs
10. $4\frac{1}{2}$ hrs
11. 100 min
12. 3
13. 14 hrs
14. 36 hrs
15. 6 hrs
16. 144 min
17. 18
18. 45
19. 14 min 40 sec
20. 12 hrs
21. x =30
22. 45 min
23. $24\frac{2}{3}$ hrs

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